Alfred Gessow Rotorcraft Center
Aerospace Engineering Department
University of Maryland, College Park

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High-Order Non-Oscillatory Compact Reconstruction Scheme for Overset Grids

Debojyoti Ghosh
Graduate Research Assistant

Shivaji Medida
Graduate Research Assistant

James D. Baeder
Associate Professor

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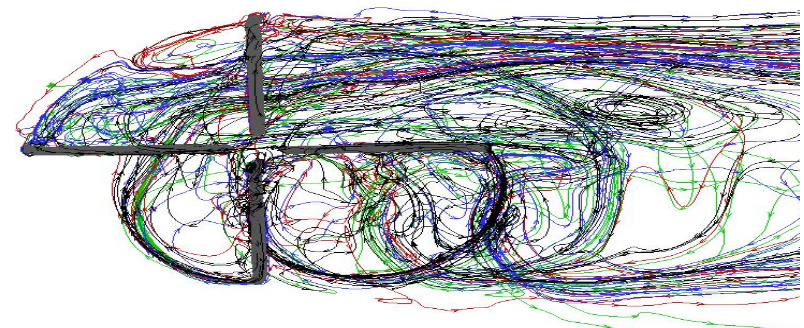
Motivation

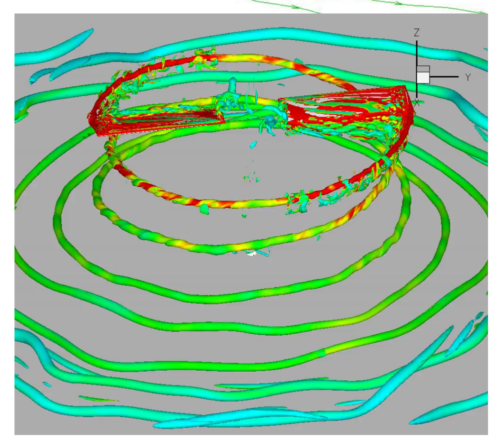
Accurate numerical simulation of the wake flow field around a rotorcraft

- Long term convection and mutual interaction of vortices
- Interactions of vortices with fuselage and ground plane
- Accurate resolution of near-blade turbulent structures



High order accurate Navier-Stokes solver

- High spectral resolution for accurate capturing of smaller length scales
- Non-oscillatory solution across shock waves and shear layers
- Low dissipation errors for preservation of flow structures over large distances



Compact-Reconstruction WENO Schemes

The **Compact-Reconstruction WENO** (CRWENO)★ scheme

- Convex combination of *r*-th order candidate *compact* interpolations
- Optimal weights in smooth regions → (*2r-1*)-th order *compact* interpolation
- Smoothness - dependent weights → Non-oscillatory interpolation for discontinuities

Smoothness Indicators

Optimal Weights

WENO Weights

$$f_{j+1/2} = \sum_{k=1}^{r} \omega_k f^k_{j+1/2}$$

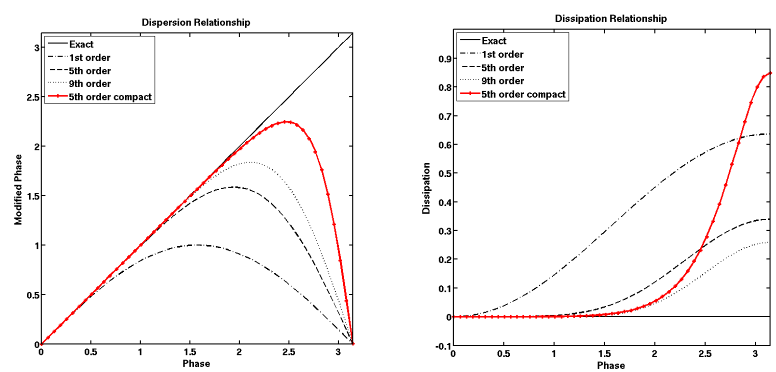
$$\alpha_k = \frac{c_k}{(\beta_k + \varepsilon)^p}; \quad \omega_k = \frac{\alpha_k}{\sum_k \alpha_k}$$

Interface flux

Candidate *compact* stencils

Why **Compact Reconstruction**?

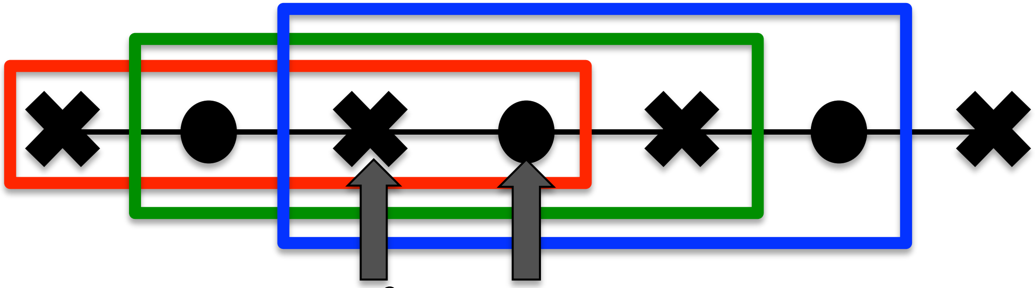
- High order accuracy with smaller stencils
- Better spectral resolution than explicit interpolation (bandwidth resolving efficiency)
- Lower dissipation at resolved frequencies
- Taylor series error order of magnitude lower



Dispersion and dissipation relationships

★ Ghosh & Baeder, Compact Reconstruction Schemes with Weighted ENO Limiting for Hyperbolic Conservation Laws, *SIAM J. Sci. Comp.*, 34(3), 2012

5^{th} Order CRWENO scheme



$$\frac{2}{3} f_{j-1/2} + \frac{1}{3} f_{j+1/2} = \frac{1}{6} f_{j-1} + \frac{5}{6} f_j \longrightarrow \cancel{c_1 = \frac{2}{10}} \quad \omega_1$$

$$\frac{1}{3} f_{j-1/2} + \frac{2}{3} f_{j+1/2} = \frac{5}{6} f_j + \frac{1}{6} f_{j+1} \longrightarrow \cancel{c_2 = \frac{5}{10}} \quad \omega_2$$

$$\frac{2}{3} f_{j+1/2} + \frac{1}{3} f_{j+3/2} = \frac{1}{6} f_j + \frac{5}{6} f_{j+1} \longrightarrow \cancel{c_3 = \frac{3}{10}} \quad \omega_3$$

$$\frac{3}{10} f_{j-1/2} + \frac{6}{10} f_{j+1/2} + \frac{1}{10} f_{j+3/2} = \frac{1}{30} f_{j-1} + \frac{19}{30} f_j + \frac{10}{30} f_{j+1}$$

$$\left(\frac{2}{3}\omega_1 + \frac{1}{3}\omega_2\right) f_{j-1/2} + \left(\frac{1}{3}\omega_1 + \frac{2}{3}(\omega_2 + \omega_3)\right) f_{j+1/2} + \frac{1}{3}\omega_3 f_{j+3/2} = \frac{\omega_1}{6} f_{j-1} + \frac{5(\omega_1 + \omega_2)}{6} f_j + \frac{\omega_2 + 5\omega_3}{6} f_{j+1}$$

Baseline Solver

Integration of the CRWENO scheme with a compressible Navier Stokes solver for overset structured meshes

- **Time Marching:** 2^{nd} order Backward Differencing (BDF2) and 3^{rd} order Total Variation Diminishing Runge Kutta (TVDRK3)
- **Dual time-stepping** for time-accurate computations
- **Implicit Inversion:** Diagonalized ADI and LU-SGS
- **Spatial reconstruction:**
  - *5^{th} order CRWENO scheme (compact)*
  - 3^{rd} order MUSCL and 5^{th} order WENO schemes (non-compact)
- **Upwinding:** Roe's flux differencing
- **Turbulence Modeling:** Spallart-Almaras one-equation model
- **Implicit hole-cutting** for overset meshes
- **Viscous Terms** discretized by 2^{nd} order central differences

Applications

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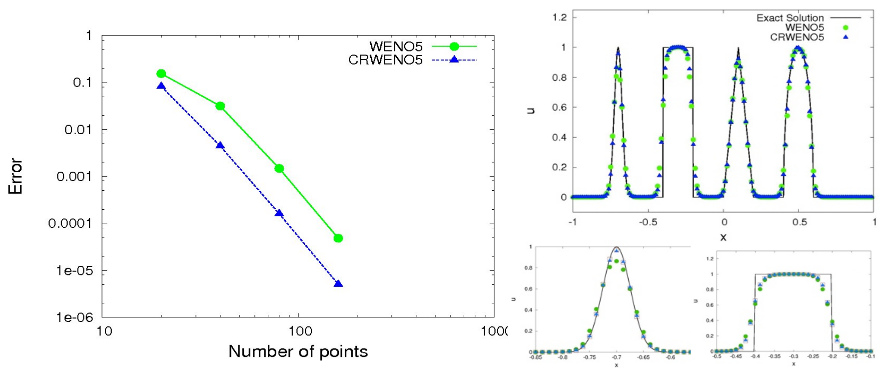
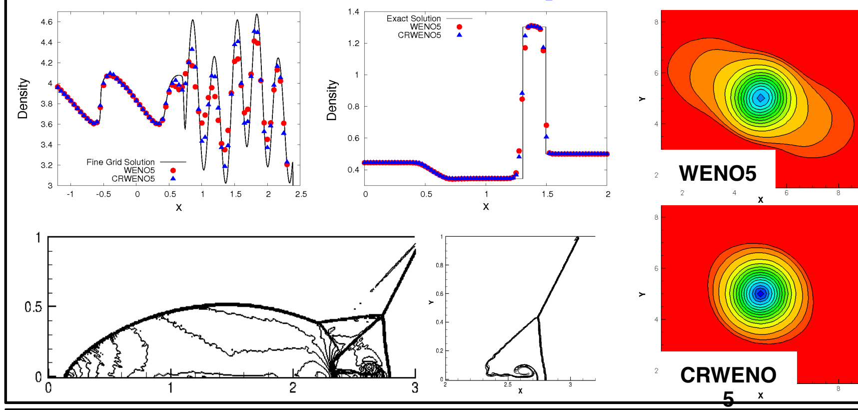
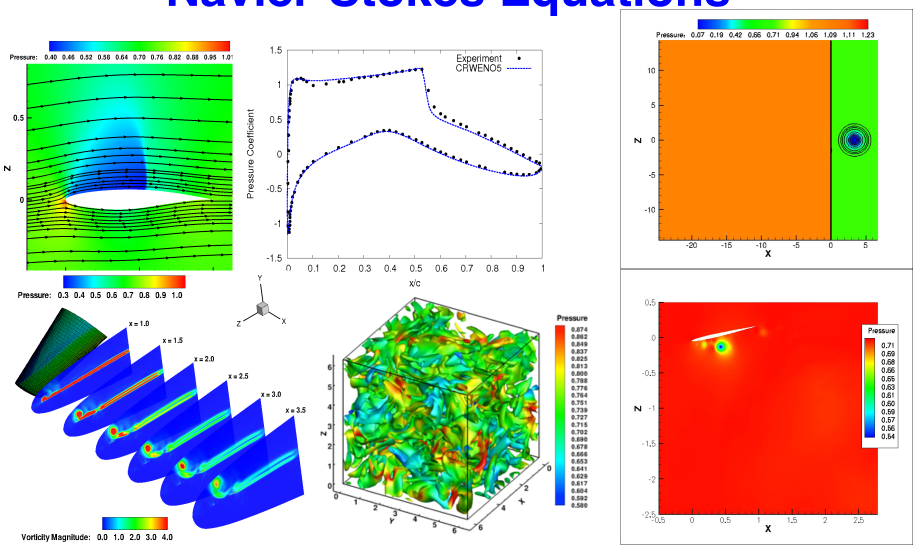
Scalar Conservation Laws



Inviscid Euler Equations



Navier-Stokes Equations

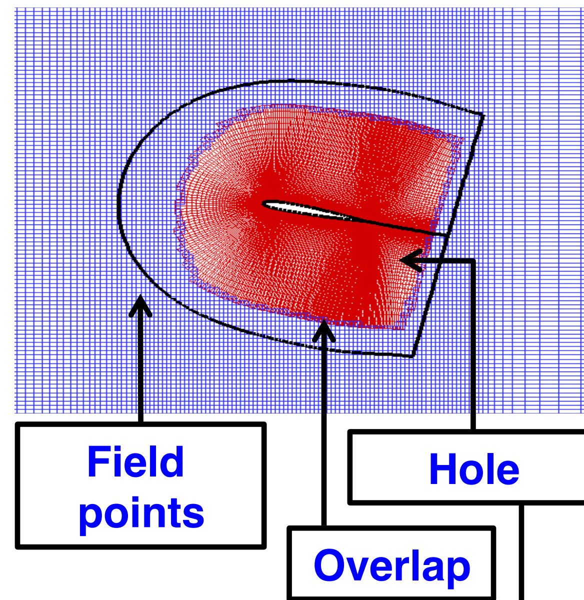


- Non-oscillatory solutions across discontinuities
- Absolute errors order of magnitude lower than WENO5 scheme
- Sharper resolution of extrema & shocks/ contact discontinuities
- Significantly lower dissipation for smaller length scales
- Improved preservation of flow structures over large convection distances
- Validated for curvilinear meshes

Overset Grids

Solution algorithm on overset meshes

- Identification of field, overlap and hole regions
- Field points → Governing equations are solved
- Overlap region → Solution exchanged with other meshes
- Hole region → Blanked out, contains non-physical values
- Implicit Hole-Cutting (Lee & Baeder, 2008)
- Tri-linear interpolation of solution between donor and receiver points



Application of compact schemes

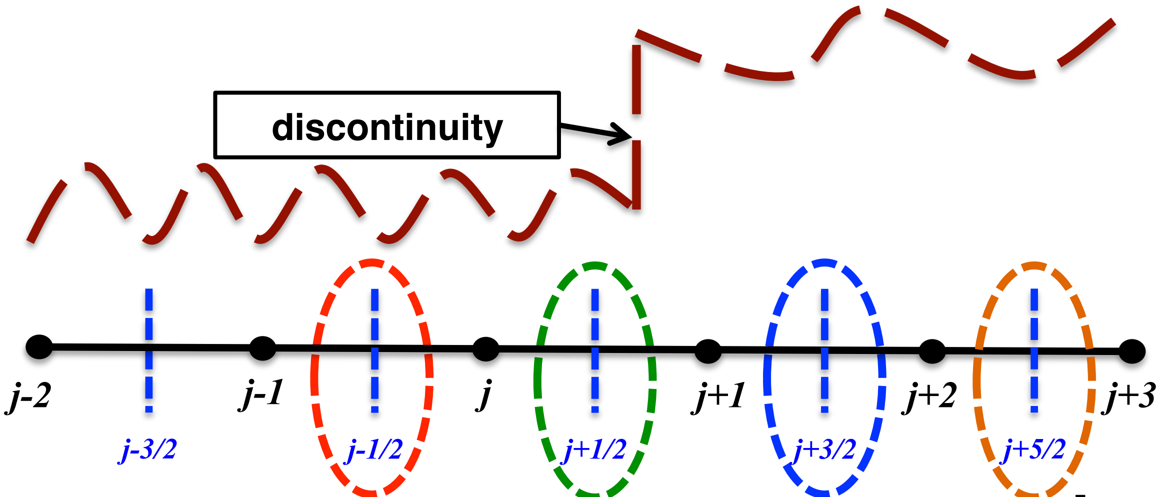
- Coupled solution for the interface fluxes
- Solution in hole region coupled with solution at field points
- System of equations contain non-physical values from the hole region

$$\begin{bmatrix} b & c & & & & & & \\ a & b & c & & & & & \\ & a & b & c & & & & \\ & & a & b & c & & & \\ & & & a & b & c & & \\ & & & & a & b & c & \\ & & & & & a & b & c \\ & & & & & & a & b \end{bmatrix} \begin{bmatrix} f_{1/2} \\ \\ \\ f_{j-1/2} \\ f_{j+1/2} \\ \\ \\ f_{N+1/2} \end{bmatrix} = \text{RHS}$$

CRWENO on Overset Grids

Behavior across discontinuity ↔ Behavior across hole cut

(Non-physical values appear as a discontinuity)



Decoupling of domain across a discontinuity

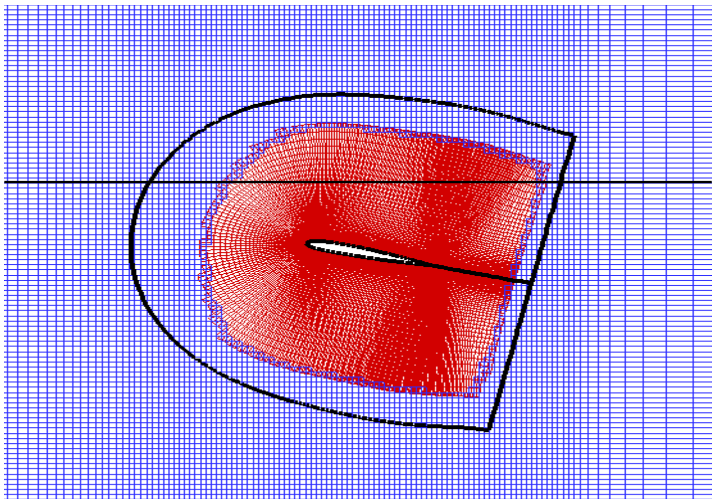
$$\frac{2}{3} f_{j-1/2} + \frac{1}{3} f_{j+1/2} = \frac{1}{6} f_{j-1} + \frac{5}{6} f_j \quad \Rightarrow \quad \omega_1 \quad ✖ \; ✖ \; 0 \; 0$$

$$\frac{1}{3} f_{j-1/2} + \frac{2}{3} f_{j+1/2} = \frac{5}{6} f_j + \frac{1}{6} f_{j+1} \quad \Rightarrow \quad \omega_2 \quad ✖ \; 0 \; 0 \; ✖$$

$$\frac{2}{3} f_{j+1/2} + \frac{1}{3} f_{j+3/2} = \frac{1}{6} f_j + \frac{5}{6} f_{j+1} \quad \Rightarrow \quad \omega_3 \quad 0 \; 0 \; ✖ \; ✖$$

$$\left[\begin{array}{cccc|cccc}
b & c & & & & & & \\
a & b & c & & & & & \\
 & a & b & c & & & & \\
 & & a & b & \mathbf{0} & & & \\
\hline
 & & & \mathbf{0} & b & c & & \\
 & & & & a & b & c & \\
 & & & & & a & b & c \\
 & & & & & & a & b
\end{array}\right]
\left[\begin{array}{c}
f_{1/2} \\ \vdots \\ f_{j+1/2} \\ \hline f_{j+3/2} \\ \vdots \\ f_{N+1/2}
\end{array}\right] = \text{RHS}$$

CRWENO on Overset Grids (Contd.)



Grid Line
($1{:}i_{max}, j$)

$$
\begin{bmatrix}
b & c & & & & & & \\
a & b & c & & & & & \\
 & a & b & 0 & & & & \\
 & & 0 & b & c & & & \\
 & & & a & b & 0 & & \\
 & & & & 0 & b & c & \\
 & & & & & a & b & c \\
 & & & & & & a & b
\end{bmatrix}
\begin{bmatrix}
f_{1/2} \\ \vdots \\ \vdots \\ f_{j-1/2} \\ f_{j+1/2} \\ \vdots \\ \vdots \\ f_{N+1/2}
\end{bmatrix}
= \mathrm{RHS}
$$

Adaptive stenciling of the CRWENO scheme

Decoupling of solution between **field** and **hole** points

Applications

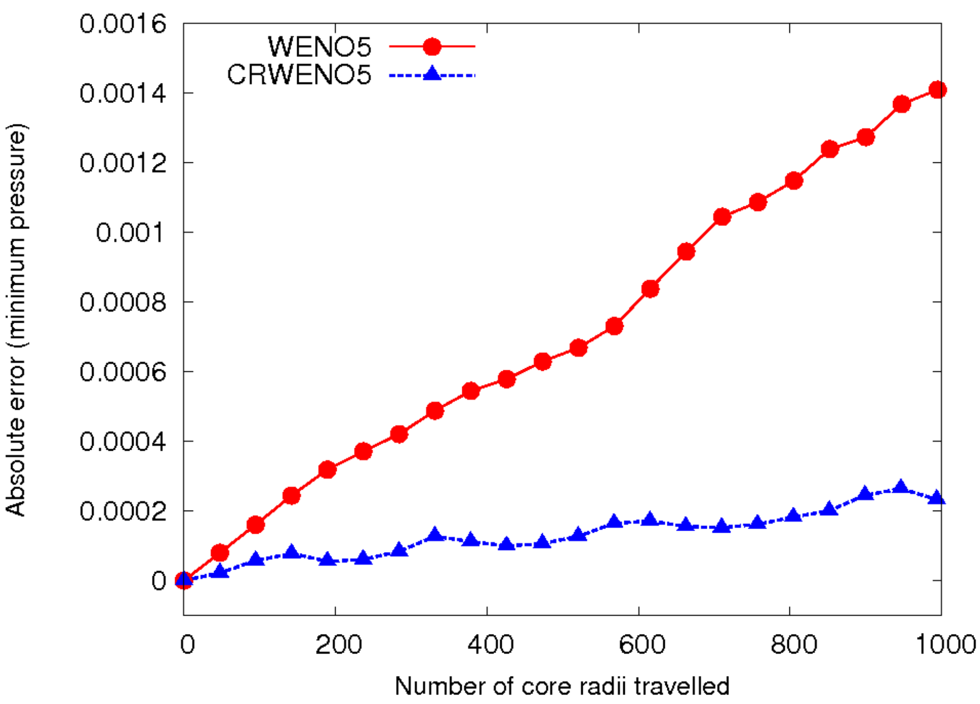
- **Verification / Validation**
  - Isentropic vortex convection
  - Steady flow over SC2110 airfoil in wind tunnel, with and without leading edge slat
  - Dynamic stall of a pitching SC1095 airfoil in wind tunnel
- **Application**
  - Flow around the Harrington two-bladed rotor

Isentropic Vortex Convection

Comparison of CRWENO5 and WENO5

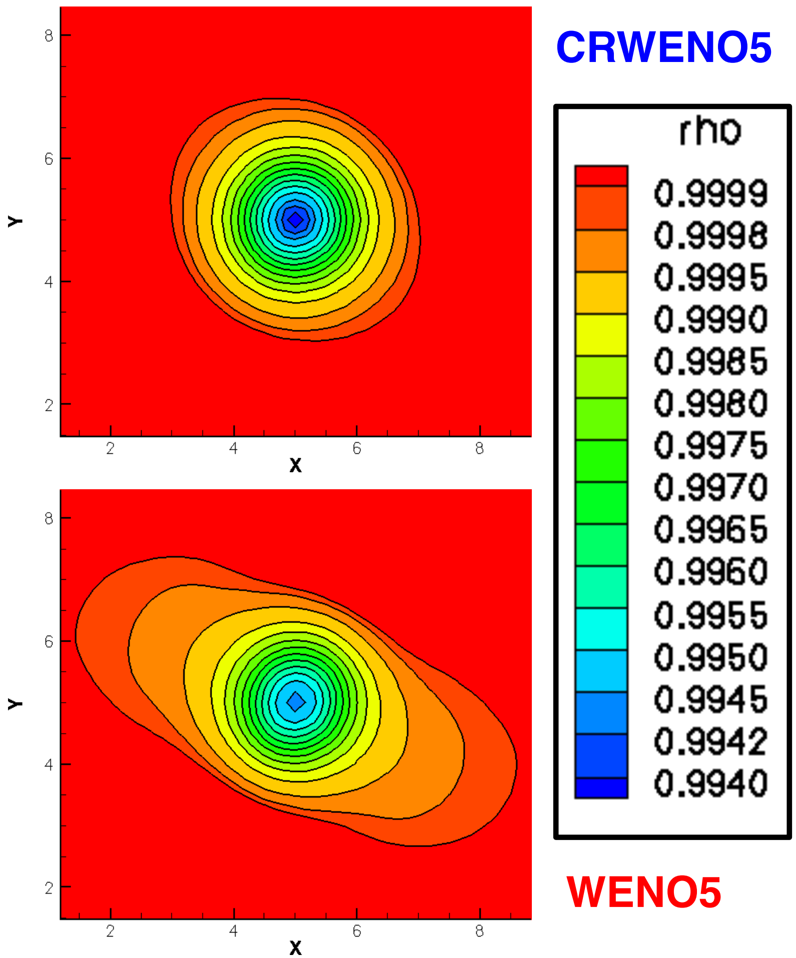
Long Term Convection (1000 core radii)

Better preservation of vortex strength and shape by the CRWENO5 scheme

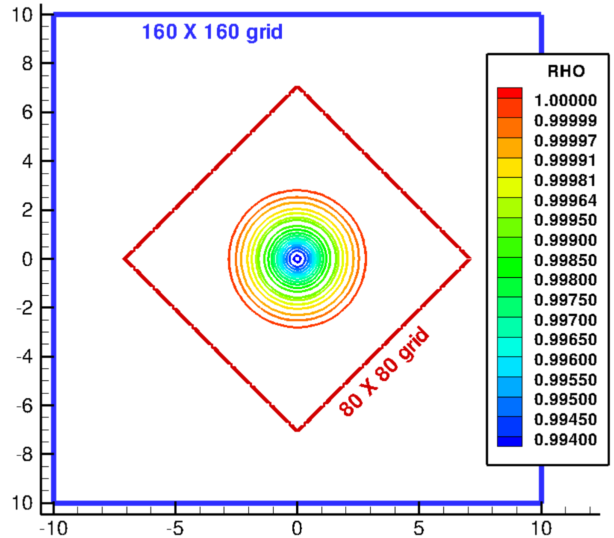


Comparison of pressure error at vortex core

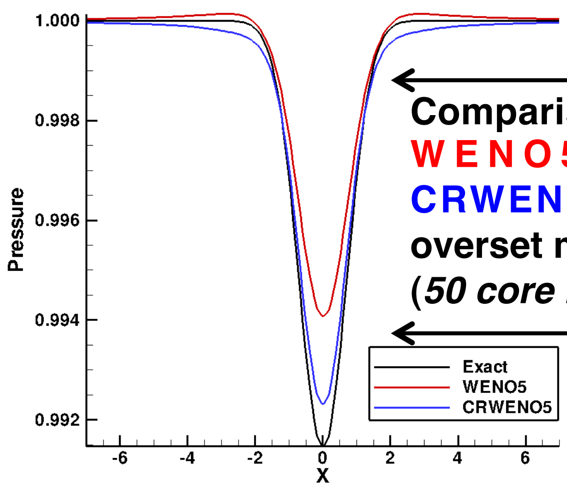
12 points across core



Isentropic Vortex Convection on Overset Grids



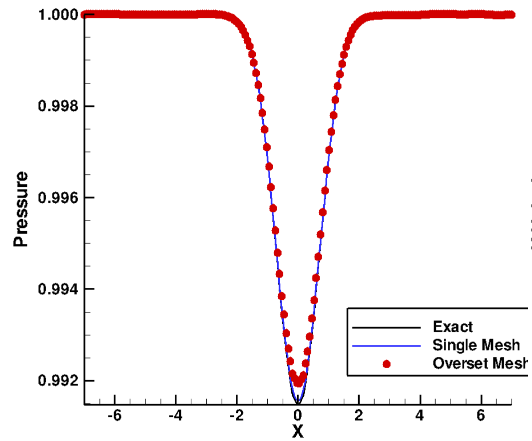
Initial conditions and domain with overset meshes

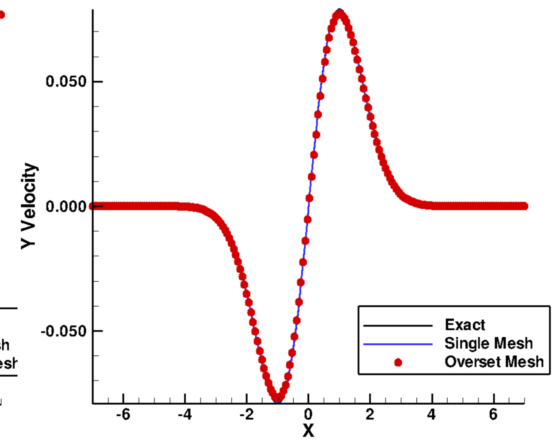


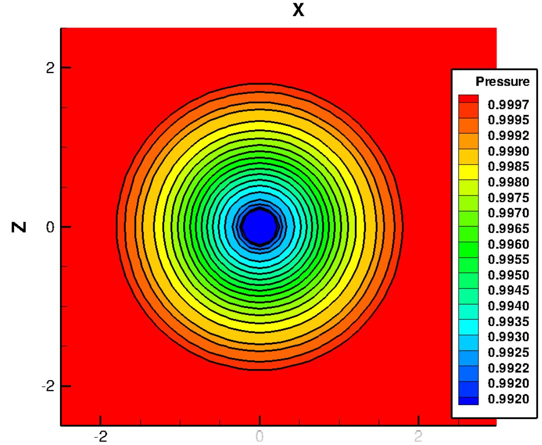
Comparison of WENO5 and CRWENO5 on overset meshes (*50 core radii*)

Comparison of solutions on single and overset meshes for the CRWENO5 scheme (*20 core radii*)

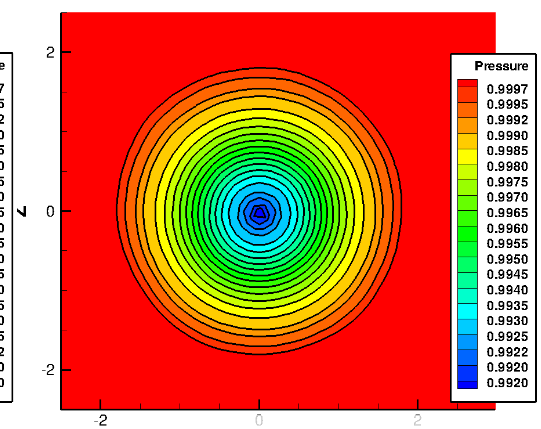
→ Good agreement





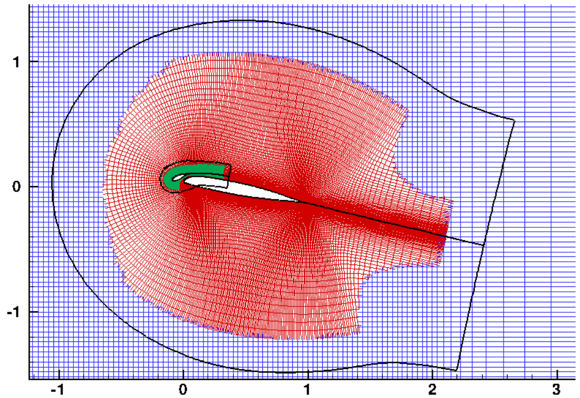


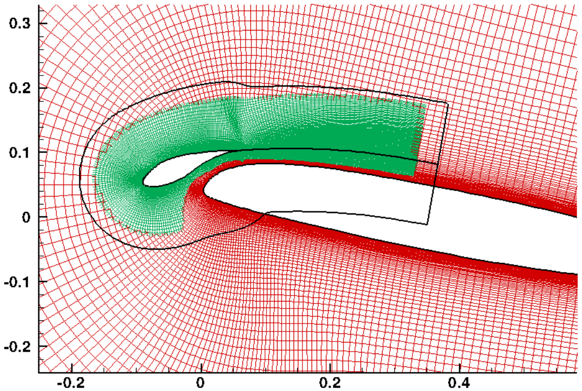
Single Mesh



Overset Mesh

SC2110 Airfoil w/ Slat in Wind Tunnel

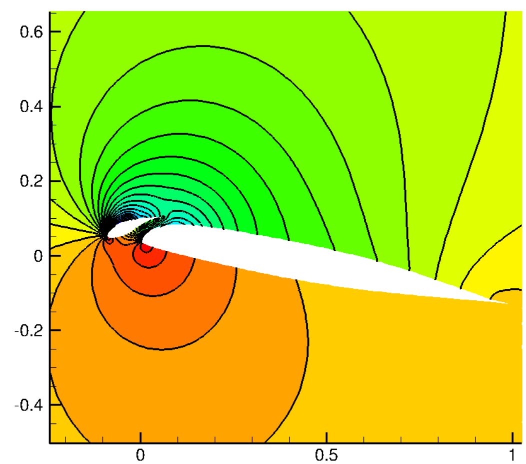




Wind Tunnel Mesh – Clustered Cartesian, 151x101 points

Airfoil Mesh – C-type, 365x138 points

Slat Mesh – C-type, 317x97 points



Pressure contours (CRWENO5 w/ BDF2)

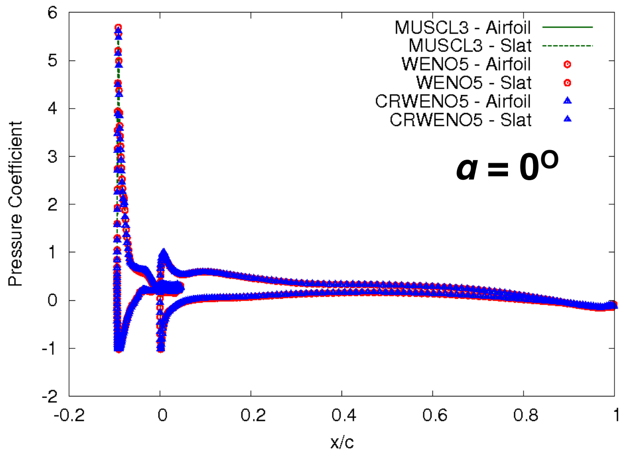
Flow Conditions:

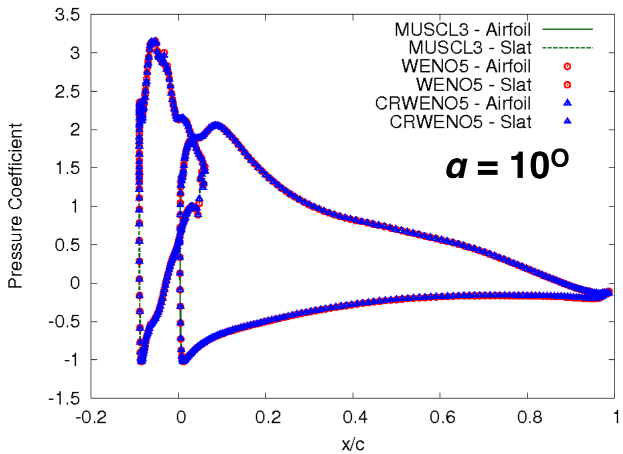
Reynolds number = 4.15e6

Freestream Mach = 0.283

Angle of Attack = 0^{o}, 10^{o}

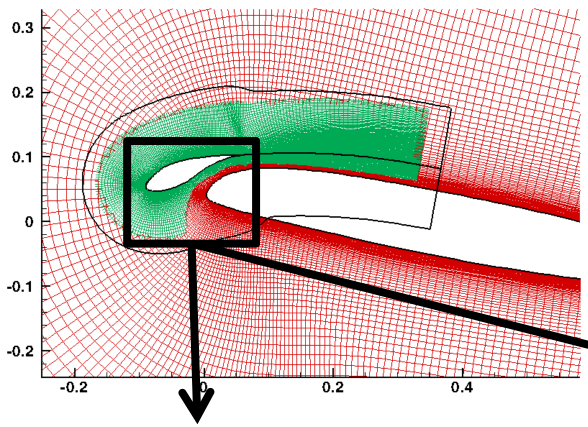
Verification of CRWENO5 scheme with non-compact MUSCL3 and WENO5 schemes

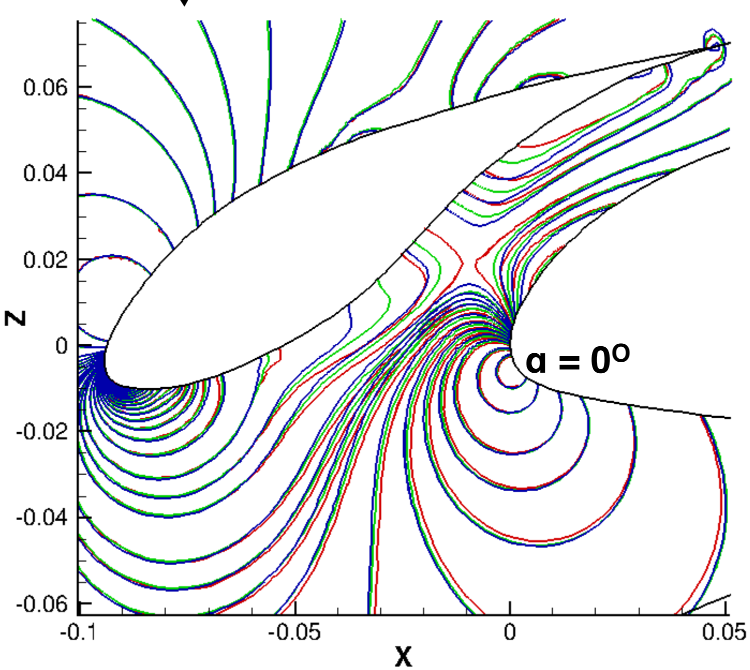


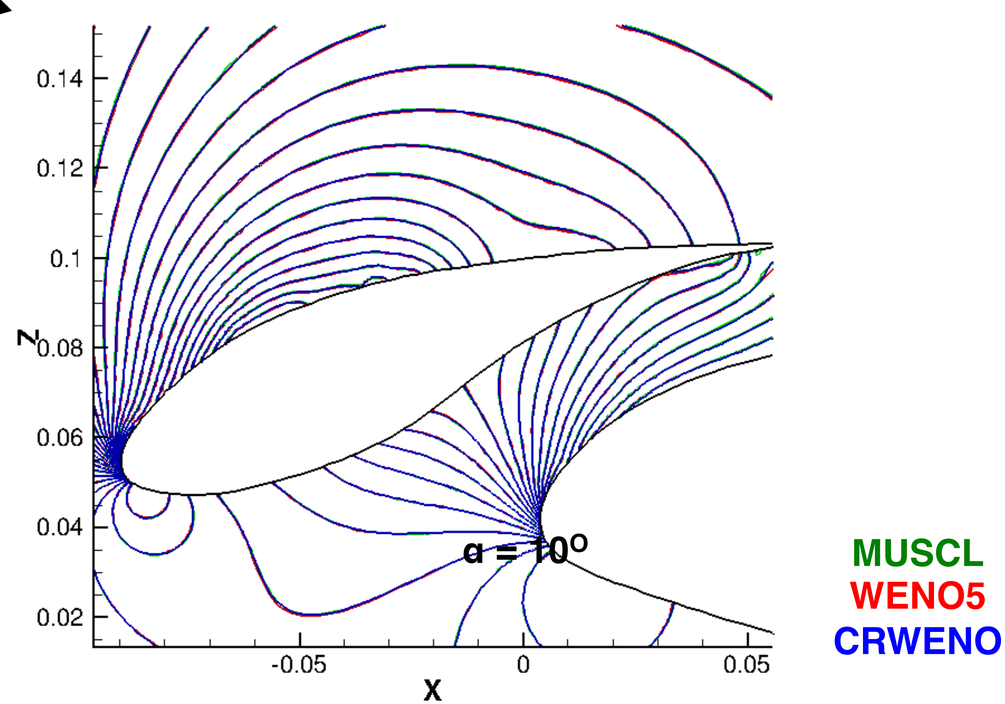


SC2110 Airfoil w/ Slat in Wind Tunnel

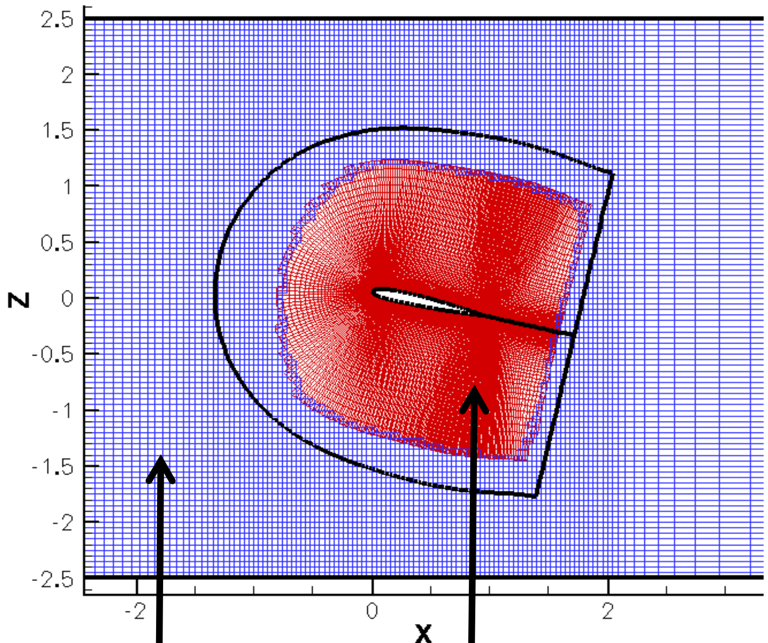
- High gradients in flow between slat and airfoil
- Overlap and solution transfer between slat and airfoil meshes
- Pressure contours from CRWENO5 compared with those from non-compact schemes





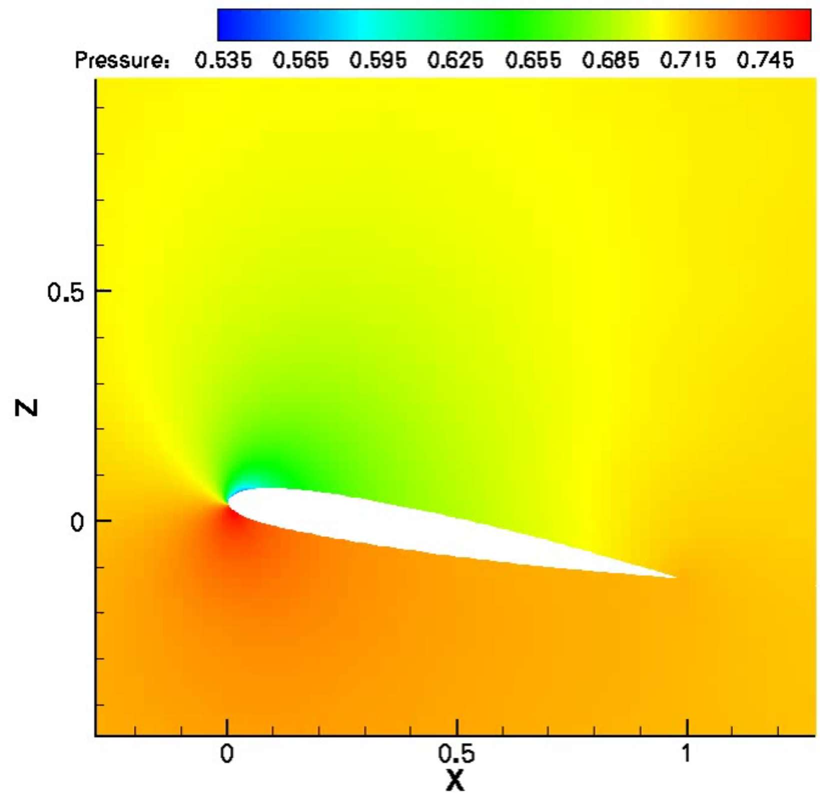


SC1095 Dynamic Stall



Airfoil Mesh – C-type, 365x138 points

Wind Tunnel Mesh – Clustered Cartesian, 151x101 points

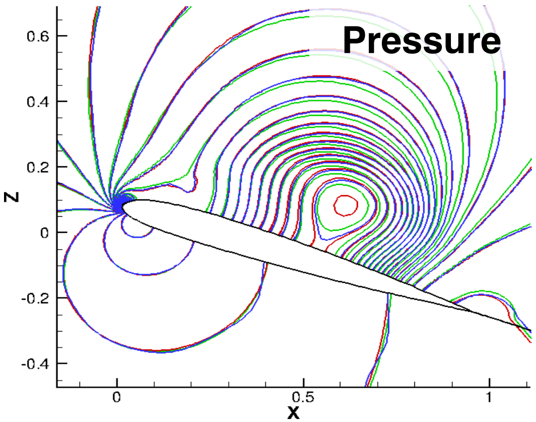


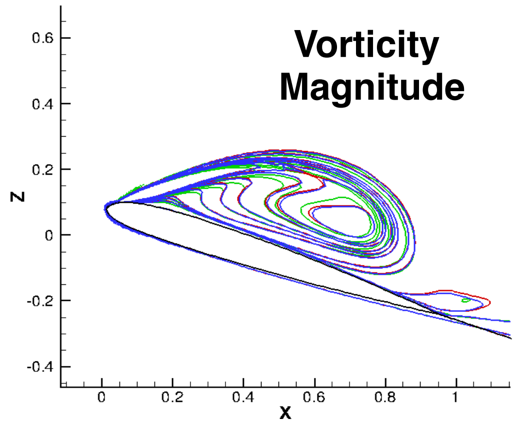
Flow Conditions:
Reynolds number: 3.92 million, Freestream Mach number 0.302 Mean angle of angle: 9.78°, Pitch Amplitude: 9.9°, Reduced Frequency: 0.099, Tunnel height: 5c

Numerical Solution: Time stepping: BDF2 w/ 15 sub-iterations

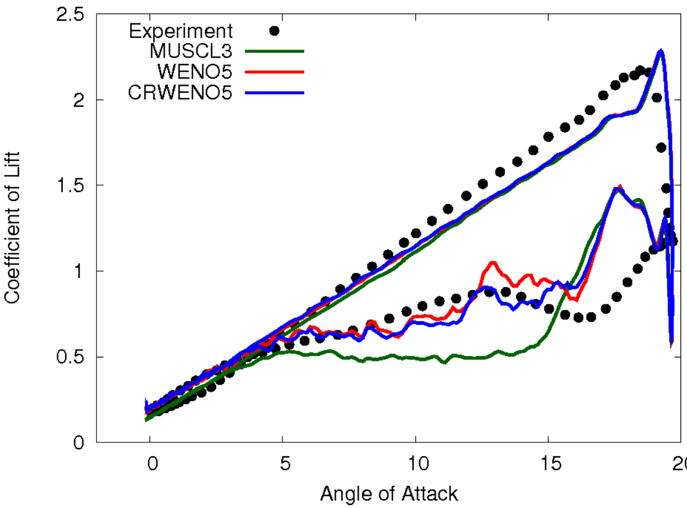
SC1095 Dynamic Stall

Validation for Overset Meshes w/ Grid Motion

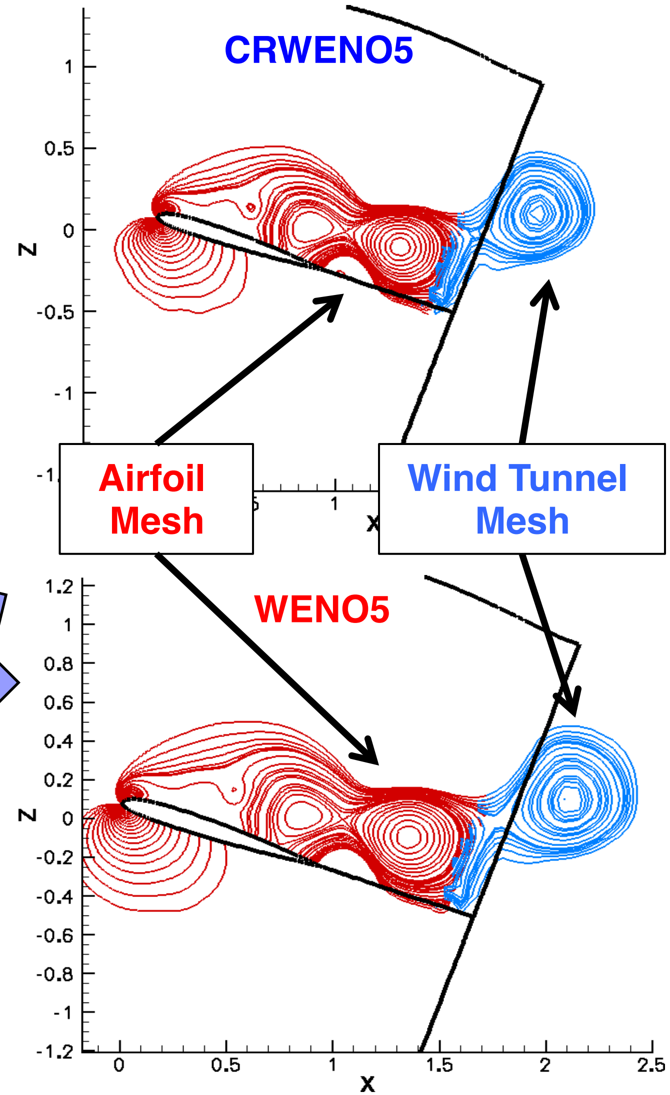




MUSCL3 WENO5 CRWENO5



Shed vortices from upper surface



- Contour lines are continuous between airfoil and wind tunnel meshes
- Shed vortices pass smoothly between the two domains

Harrington 2-Bladed Rotor

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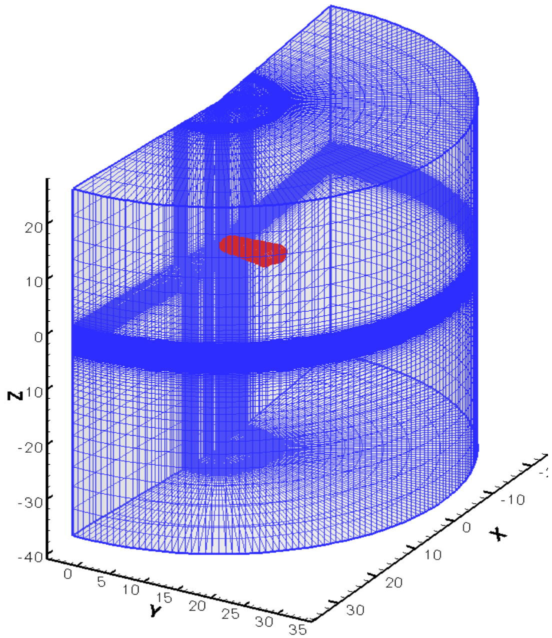
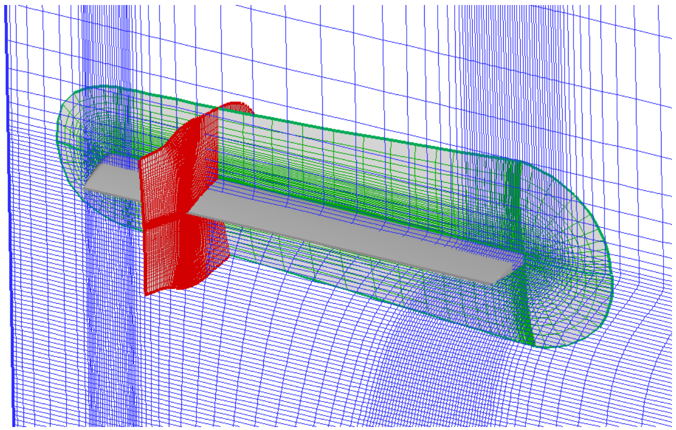




Blade Mesh – C-O type,
267x78x56 points

Flow Conditions:

- M_{tip}: 0.352
- Re_{tip}: 3.5 million

Validation of thrust & power coefficients and figure of merit

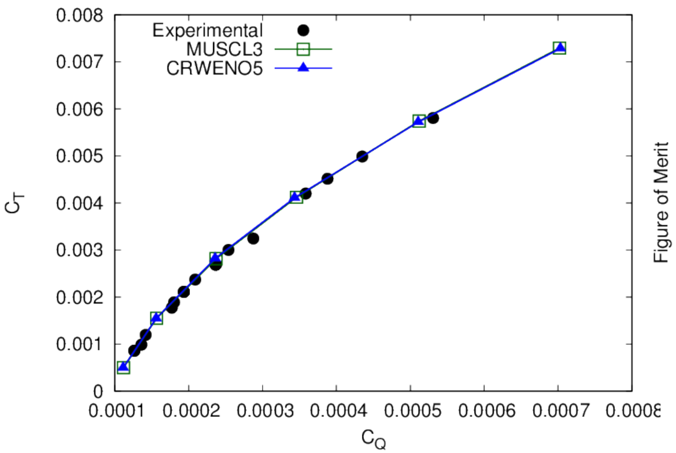
Cylindrical Back-ground Mesh

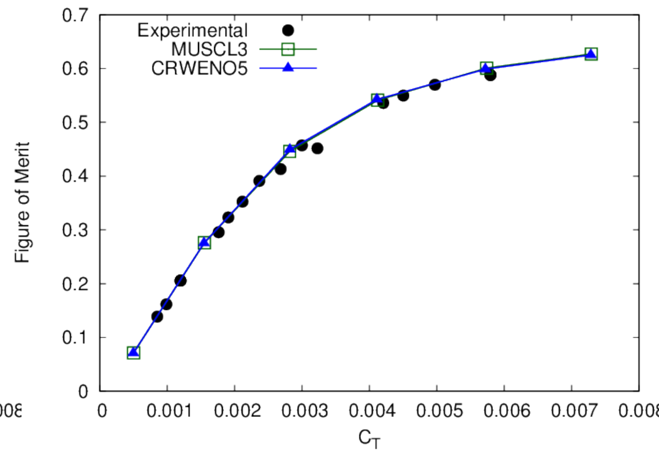
127 x 116 x 118 points

Rotor Geometry:

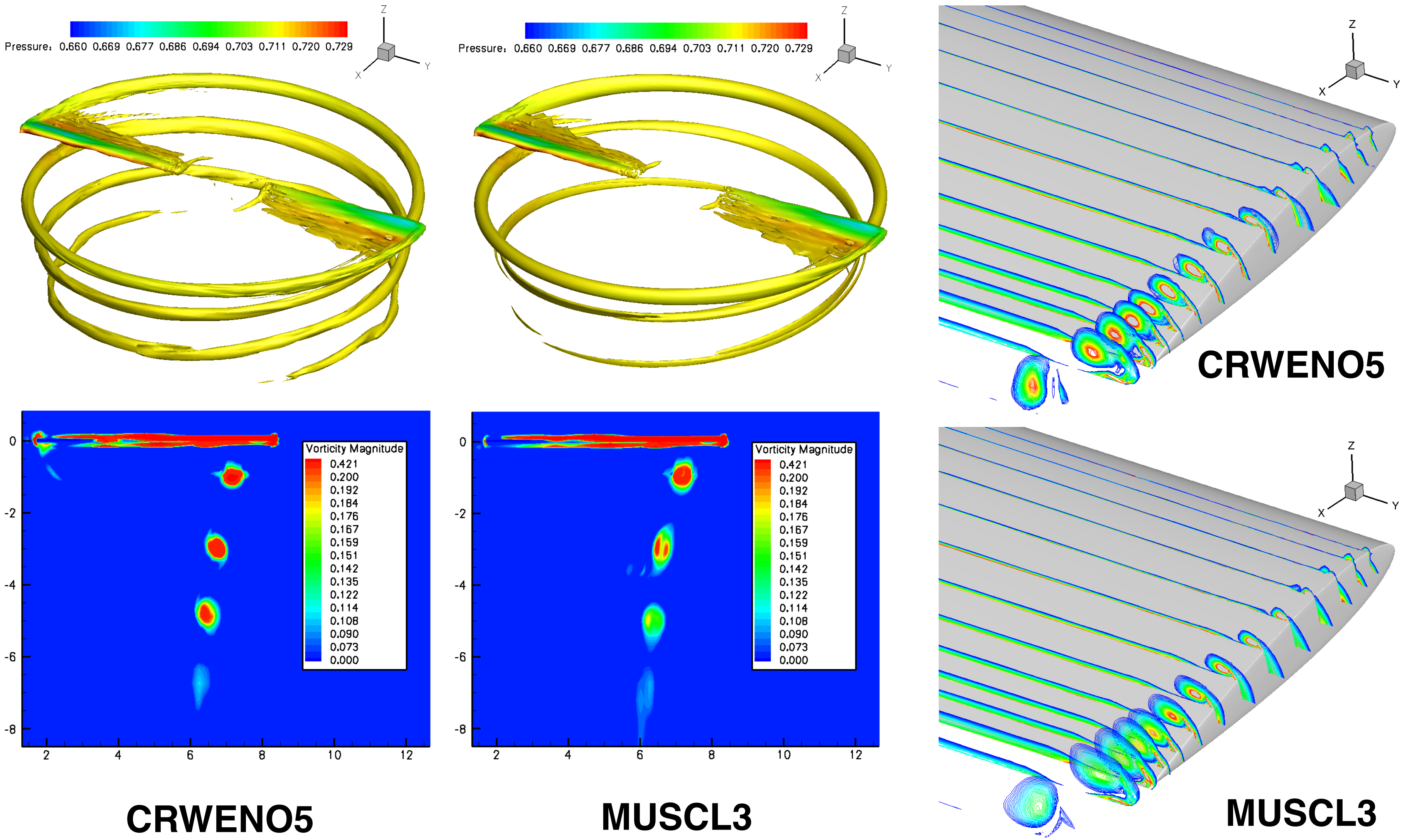
- Aspect Ratio – 8.33
- Airfoil section – NACA

(t/c: 27.5% @ 0.2R, 15% @ R)





Harrington 2-Bladed Rotor (Near-Blade and Wake Flowfield)



Conclusions and Future Work

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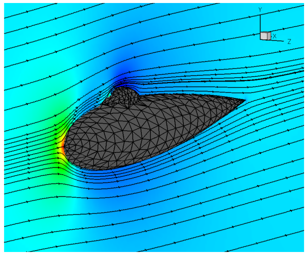
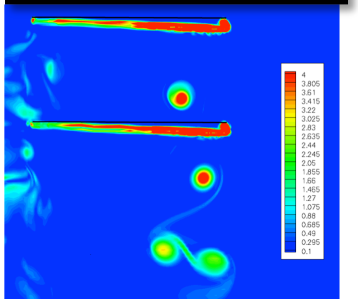
CRWENO5 scheme validated and verified for overset grids

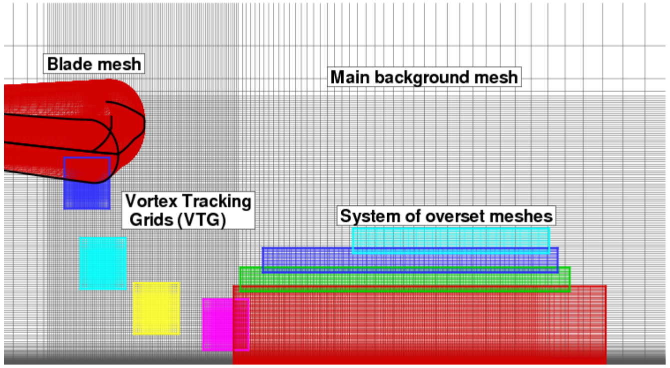
- Improved resolution of flow features due to lower numerical errors
- Slight loss of accuracy due to 2nd order interpolation between meshes
- Non-physical solution in “hole” does not pollute field solution
- Smooth transfer of solution between different grids

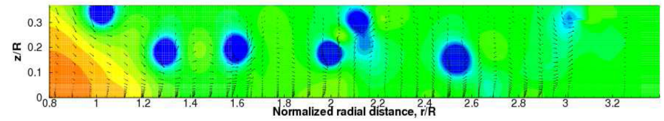




Future Work & Applications of CRWENO5

- Application to meshes w/ immersed boundaries
- Wake flow from coaxial configurations
- Rotorcraft wake flow when operating “in-ground-effect” (IGE)
- Accurate modeling of wake vortex interactions with ground plane
- Application of CRWENO5 scheme with Vortex-Tracking Grids (VTGs)
- Sound generation due to blade vortex interaction (BVI) for rotor in forward flight





Thank You!

Questions?