Chimera Overset Method with a Discontinuous Galerkin Discretization

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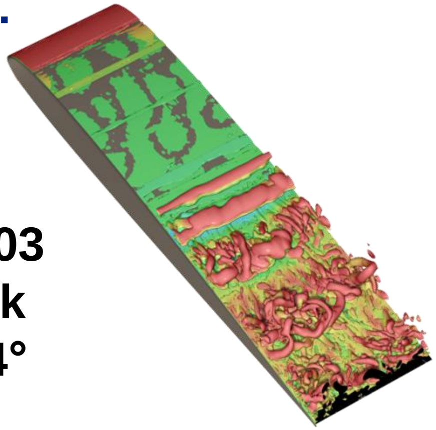
Motivation

- Chimera Overset Grid Method
  - Complex Geometries
  - "Hot swap" Geometric Features
  - Moving Grids with Relative Motion
    - Store Separation
    - Rotorcraft
- High-Order Methods
  - DNS-LES
  - Transitional Turbulent Flows
  - And more…

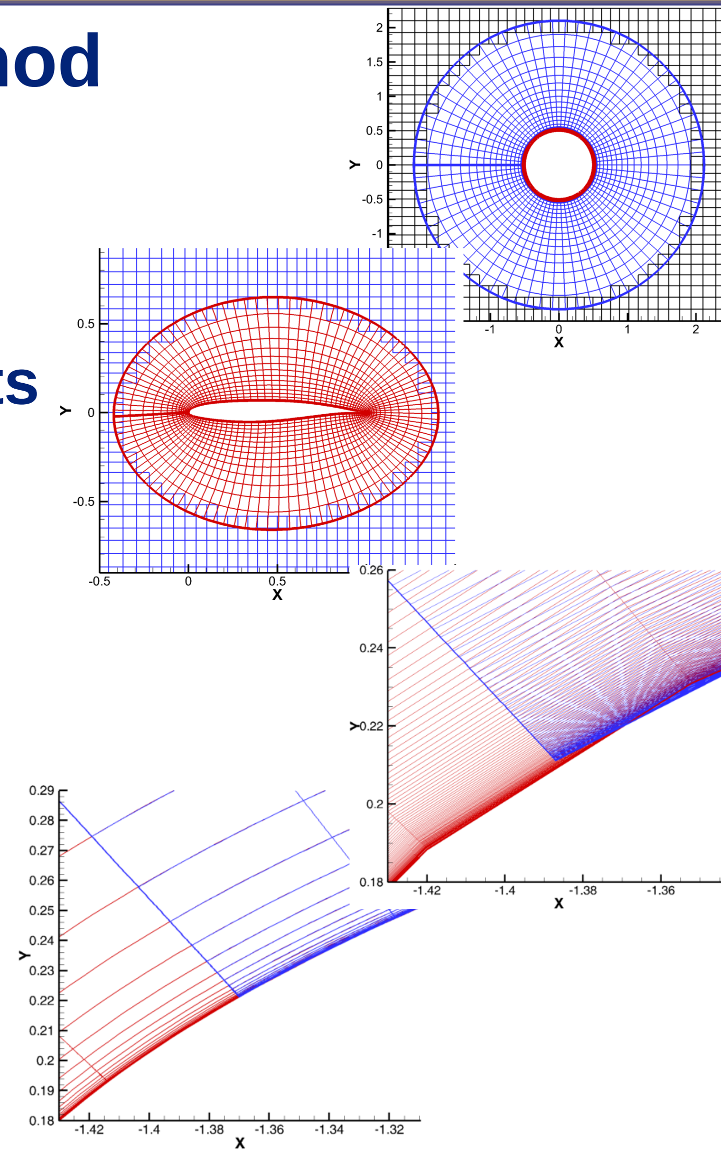


SD7003
Re 60k
AoA 4°

- WENO, Compact FD
  - Large Interior Stencils
  - Fringe Points
    - Maintain Interior Scheme
    - Orphan Points
  - High-Order Interpolation Schemes
    - Large Stencil
  - Complicated Hole Cutting
- Discontinuous Galerkin Method
  - Natural Higher Order Extension to Finite Volume
  - Compact Stencil
    - Communication
      - No Fringe Points
    - Hole Cutting
  - Curved Elements

Outline

- **Discontinuous Galerkin Method**
  - **Mesh Representation**
- **Inter-Grid Communication**
  - **Finite Volume and Fringe Points**
  - **DG-Chimera**
  - **Inviscid Flow Examples**
- **Overset Regions on Curved Geometry**
  - **Viscous Flow Examples**
- **Conclusion and Future Work**



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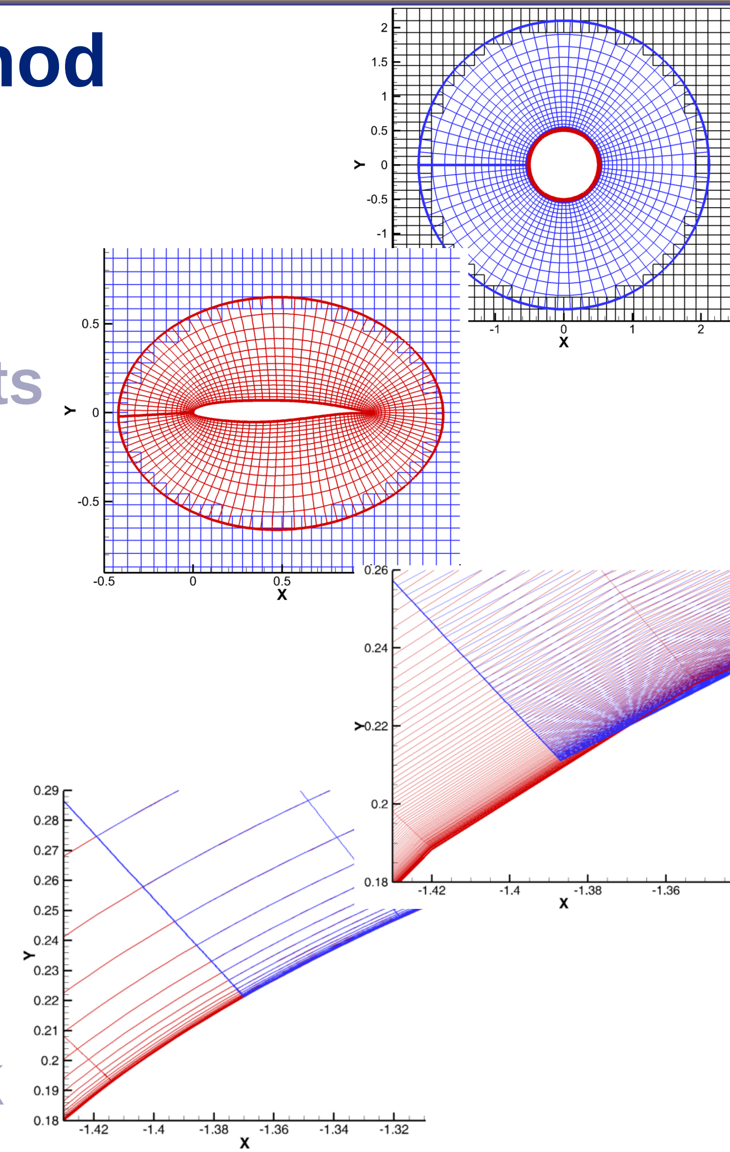
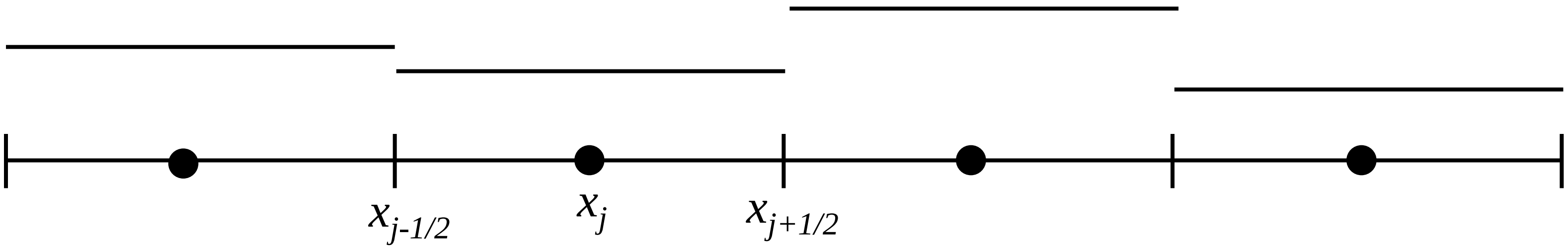


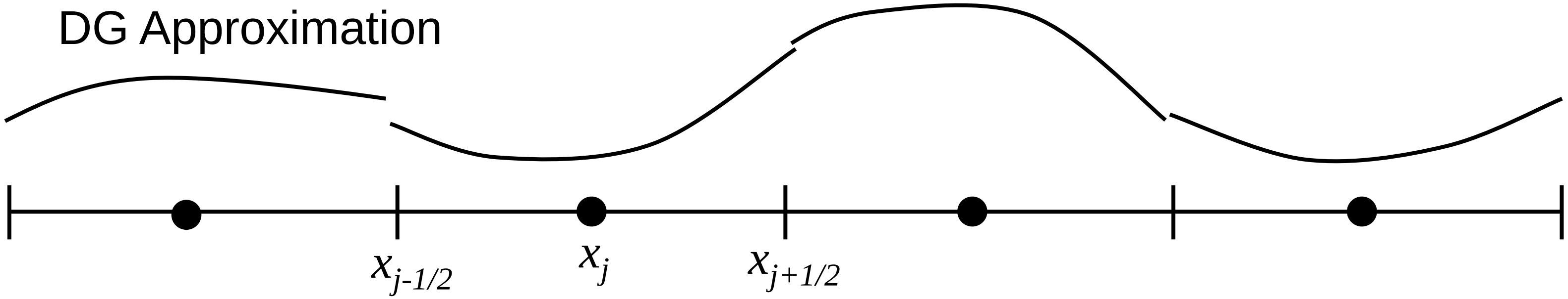
Discontinuous Galerkin Method

FV Approximation



Discontinuous Galerkin Method

DG Approximation



- **Euler/Navier-Stokes Equations in Conservation Form**

$$\nabla \cdot \vec{F} = 0$$

- **Weak Form**

$$\int_{\Omega_e} \phi \nabla \cdot \vec{F} \, d\Omega = 0$$

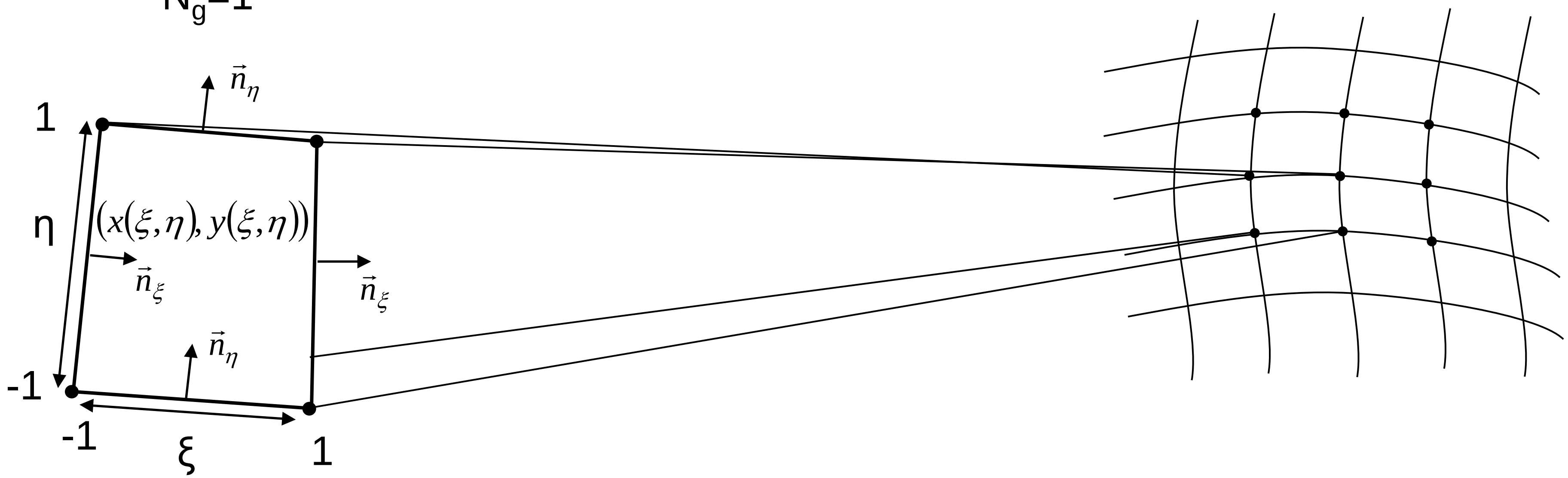
ϕ - Legendre Polynomials

$$R\left(Q^+, Q^-\right) = \int_{\Gamma_e} \phi \vec{F}\left(Q^+, Q^-\right) \cdot \vec{n} \, d\Gamma - \int_{\Omega_e} \nabla \phi \cdot \vec{F}\left(Q^-\right) d\Omega = 0$$

- **Approximate Riemann Solver by Roe**
- **BR2 Viscous Scheme**
- **Newton-Krylov Solver with GMRES and ILU1 Preconditioner**

Discontinuous Galerkin Method
Geometric Mapping

$N_g=1$



N_g – Degree of Cell Polynomial

$$x(\xi,\eta)=\sum_{i=0}^{N_g}\sum_{j=0}^{N_g} x_{ij}\phi_i(\xi)\phi_j(\eta)$$

$$y(\xi,\eta)=\sum_{i=0}^{N_g}\sum_{j=0}^{N_g} y_{ij}\phi_i(\xi)\phi_j(\eta)$$

$$\xi_x = Jy_\eta \qquad \xi_y = -Jx_\eta$$

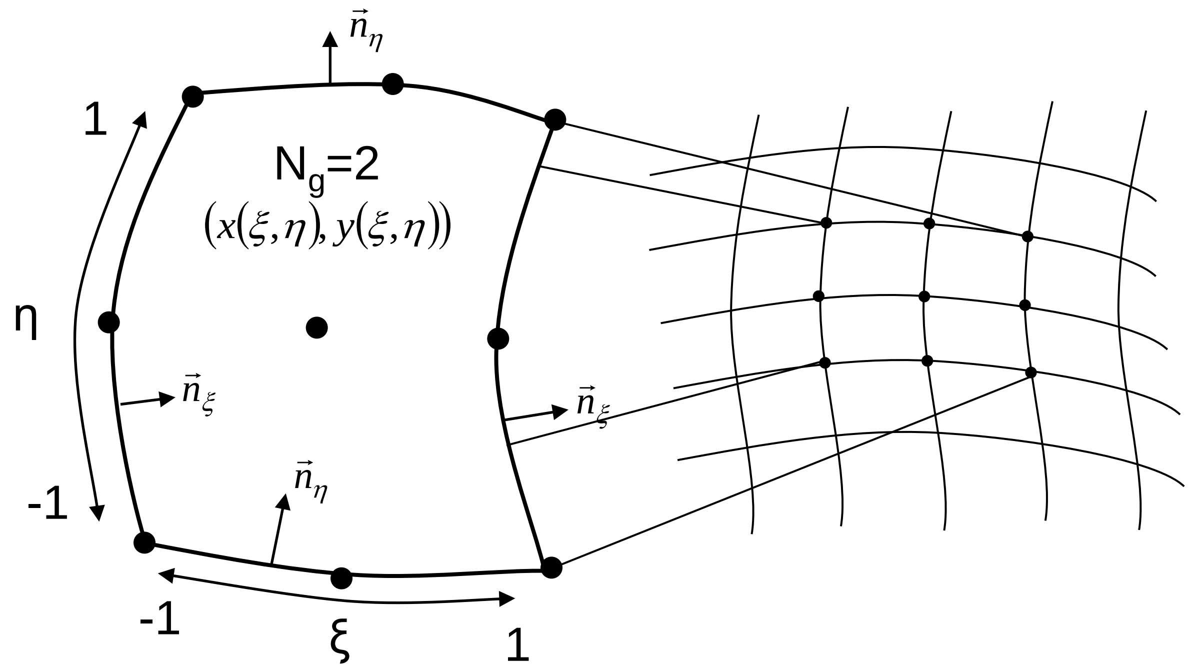
$$\eta_x = -Jy_\xi \qquad \eta_y = Jx_\xi$$

$$\frac{1}{J} = x_\xi y_\eta - x_\eta y_\xi$$

$$\vec{n}_\xi = \frac{\nabla\xi}{J} = \frac{1}{J}\left(\xi_t \quad \xi_x \quad \xi_y\right)$$

$$\vec{n}_\eta = \frac{\nabla\eta}{J} = \frac{1}{J}\left(\eta_t \quad \eta_x \quad \eta_y\right)$$

Discontinuous Galerkin Method
Geometric Mapping



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Outline

- Discontinuous Galerkin Method
  - Mesh Representation
- **Inter-Grid Communication**
  - **Finite Volume and Fringe Points**
  - **DG-Chimera**
  - **Inviscid Flow Examples**
- Overset Regions on Curved Geometry
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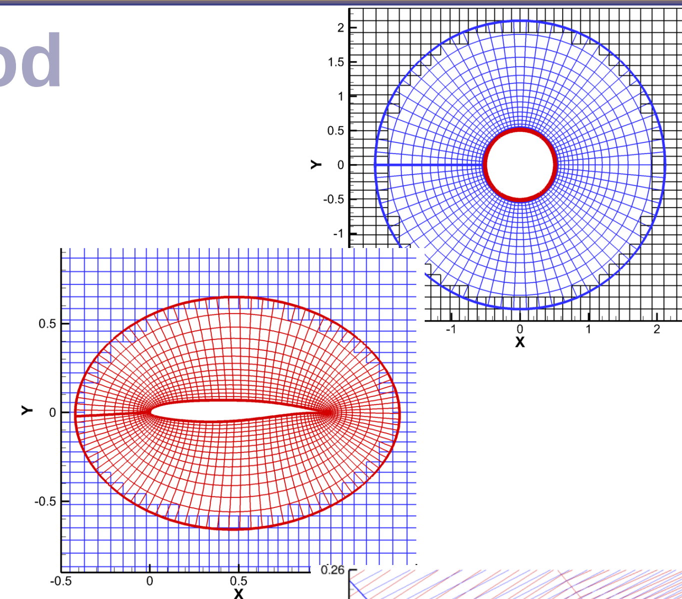
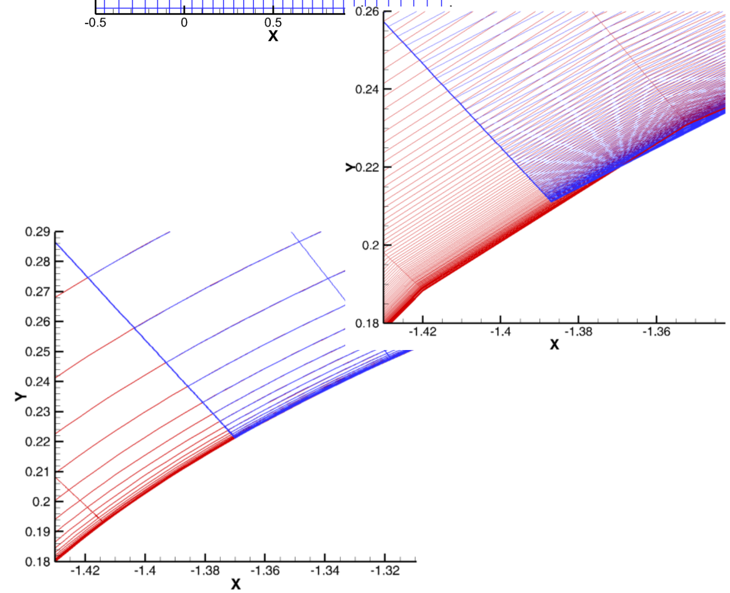
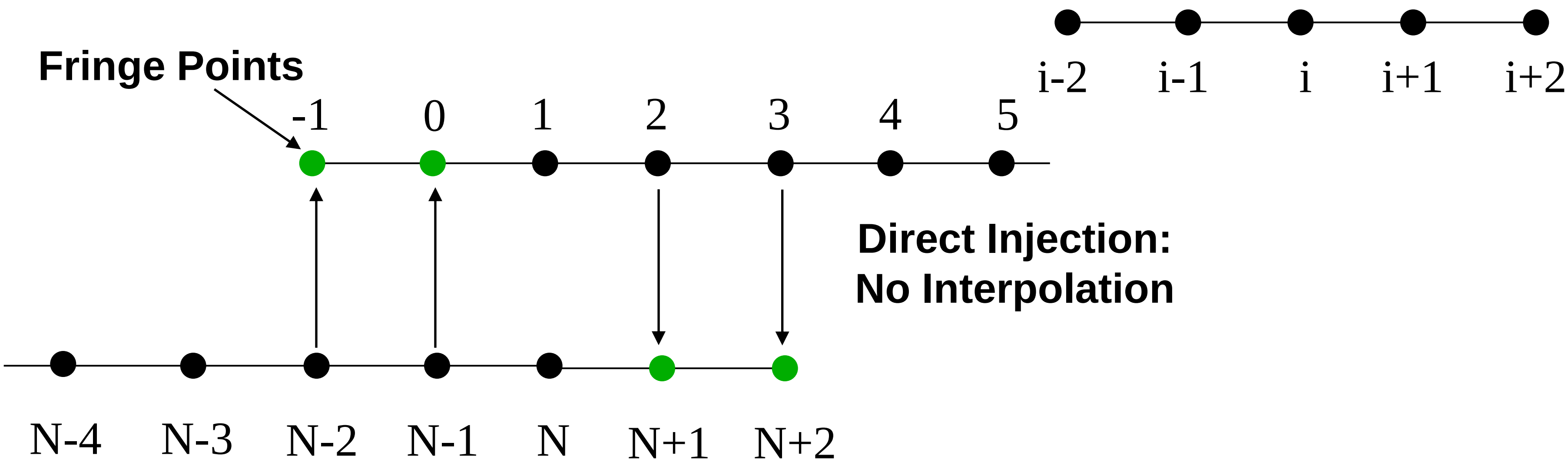




High-Order FD/FV Chimera Fringe Points

Compact Difference 6th-order

$$\frac{1}{3}\left(\frac{\partial u}{\partial \xi}\right)_{i-1} + \left(\frac{\partial u}{\partial \xi}\right)_{i} + \frac{1}{3}\left(\frac{\partial u}{\partial \xi}\right)_{i+1} = \frac{14}{9}\left(\frac{u_{i+1}-u_{i-1}}{2}\right) + \frac{1}{9}\left(\frac{u_{i+2}-u_{i-2}}{4}\right)$$



- **Fringe Points:**
  - **Additional Points on the Boundary to Maintain Interior Scheme**

High-Order FD/FV Chimera Fringe Points

Compact Difference 6th-order

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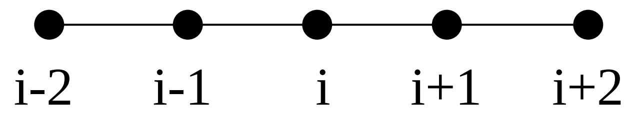

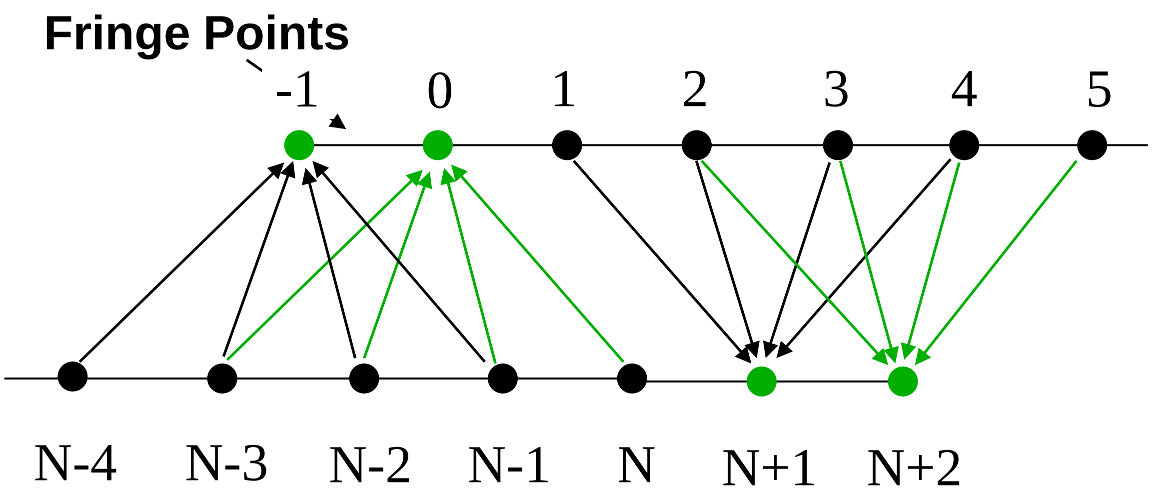

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High-Order FD/FV Chimera Fringe Points

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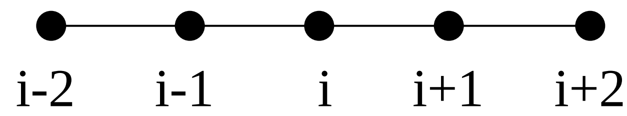
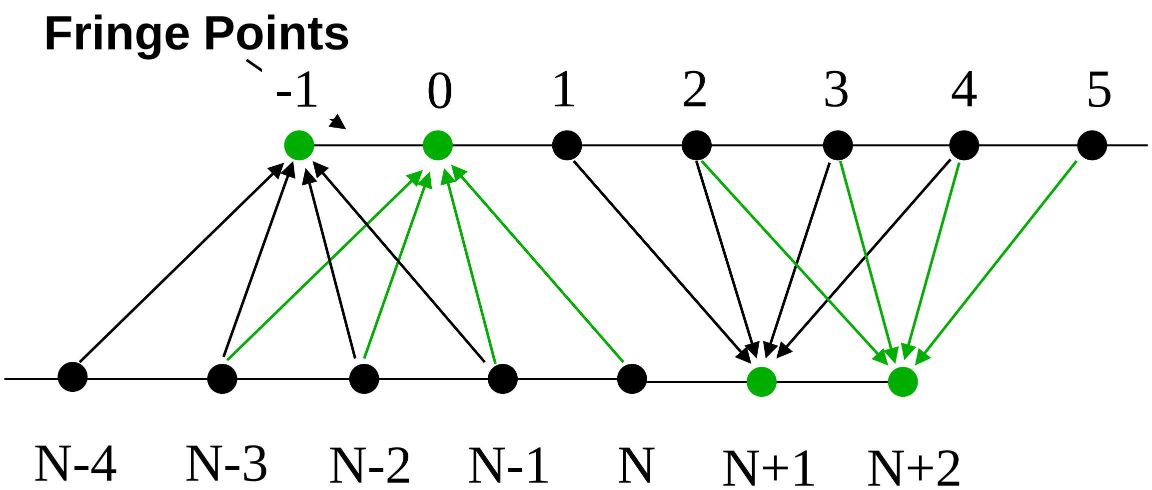
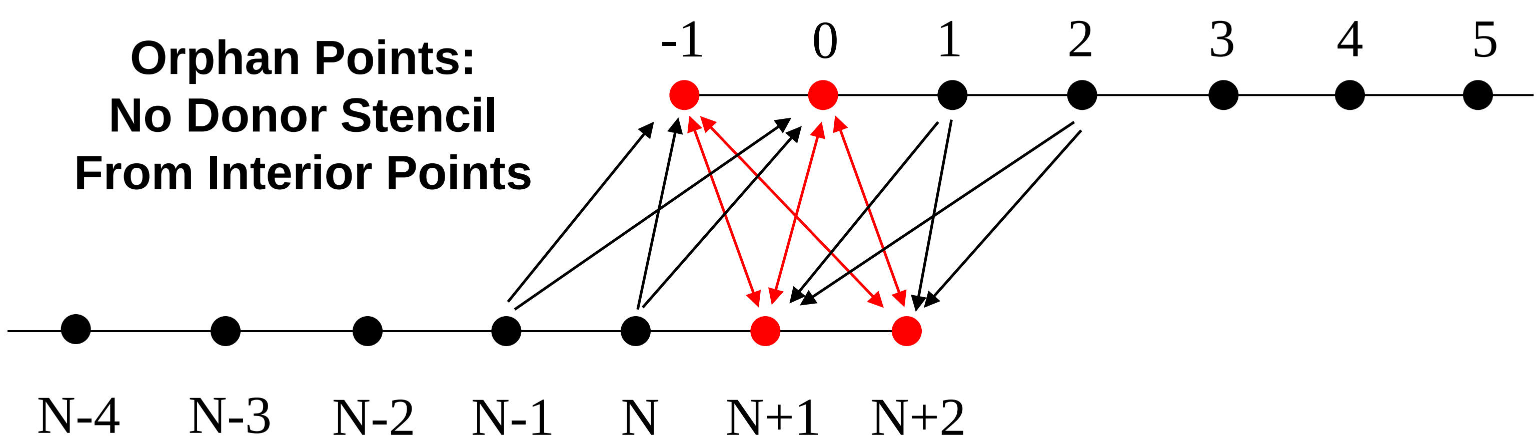
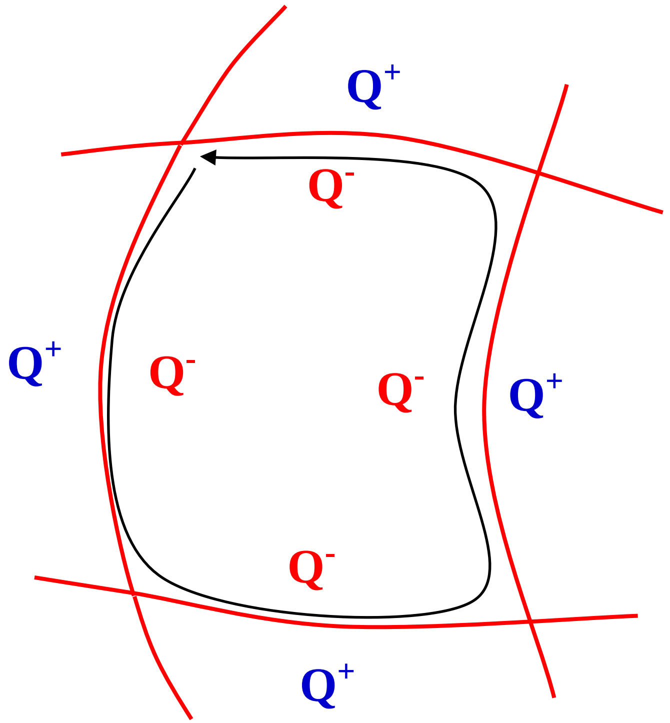




Orphan Points:
No Donor Stencil
From Interior Points



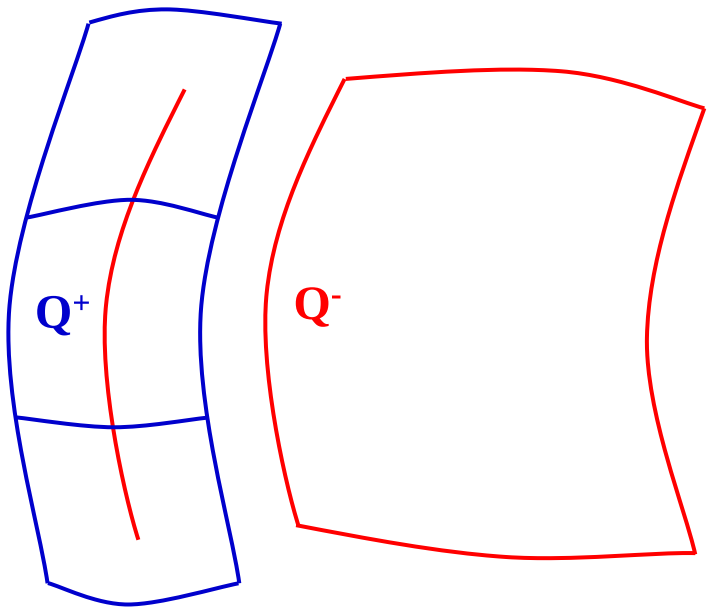
DG-Chimera
Inter-Grid Communication



$$R\left(Q^{+},Q^{-}\right)=\int_{\Gamma_e}\phi\vec{F}\left(Q^{+},Q^{-}\right)\cdot\vec{n}\,d\Gamma$$

$$-\int_{\Omega_e}\nabla\phi\cdot\vec{F}\left(Q^{-}\right)d\Omega=0$$

DG-Chimera
Inter-Grid Communication



Need $Q^{+}(\eta)=\sum_{n}^{N} q_n^{+}\phi_n(\eta)$

$$R(Q^{+},Q^{-})=\int_{\Gamma_e}\phi\vec{F}(Q^{+},Q^{-})\cdot\vec{n}\,d\Gamma - \int_{\Omega_e}\nabla\phi\cdot\vec{F}(Q^{-})\,d\Omega=0$$

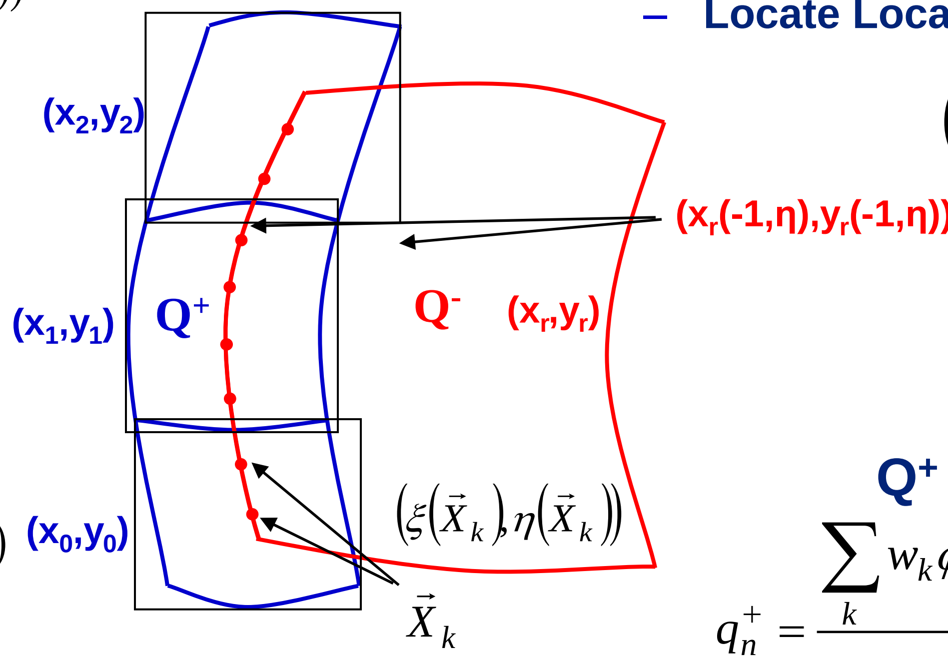
DG-Chimera
Inter-Grid Communication

Cartesian Gauss Quadrature Nodes

$$\vec{X}_k = \left(x_{r-}(s_k), y_{r-}(s_k)\right)$$

- **KD-Tree**
  - Locate Node X_k In Bounding Box
- **Newton's Method**
  - Locate Local Cell Coordinate

$$\left(\xi(\vec{X}_k), \eta(\vec{X}_k)\right)$$



Face Polynomial

$$x_{r-}(\eta) = x_r(-1,\eta) = \sum_{k=0}^{N_g} \hat{x}_k \phi_k(\eta)$$

$$y_{r-}(\eta) = y_r(-1,\eta) = \sum_{k=0}^{N_g} \hat{y}_k \phi_k(\eta)$$

Q+ Coefficients

$$q_n^+ = \frac{\sum_k w_k \phi_n(s_j) Q_b\left(\xi(\vec{X}_k), \eta(\vec{X}_k)\right)}{\int_\Gamma \phi_n^2 d\Gamma}$$

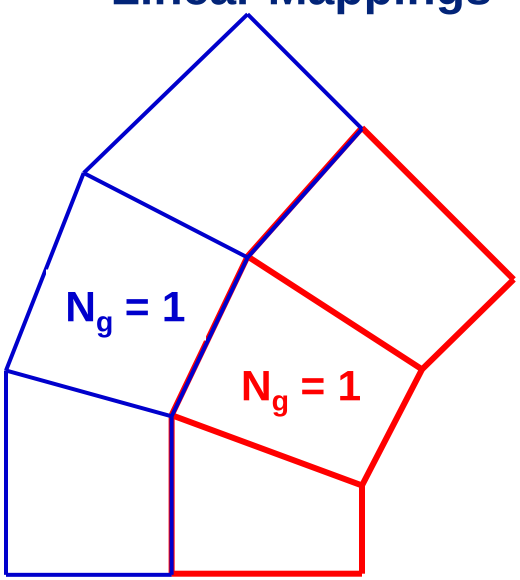
Need $Q^+(\eta) = \sum_n^N q_n^+ \phi_n(\eta)$

$$R(Q^+, Q^-) = \int_{\Gamma_e} \phi \vec{F}(Q^+, Q^-) \cdot \vec{n} d\Gamma - \int_{\Omega_e} \nabla \phi \cdot \vec{F}(Q^-) d\Omega = 0$$

DG-Chimera
Zonal Interfaces

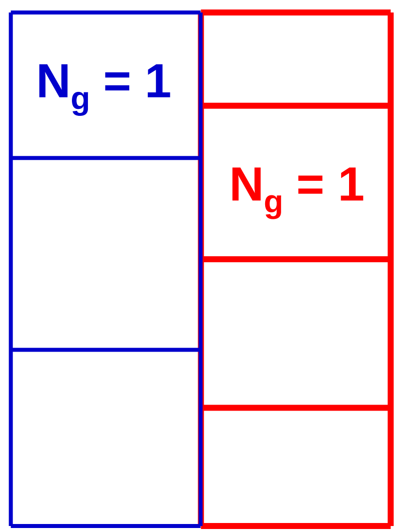
- No Additional Cells to Facilitate Communication
  - No Fringe Points
  - No Orphan Points Associated with Fringe Points

- Reduces to Zonal Interface
  - Curved Boundary
  - Coincident Nodes
  - Linear Mappings

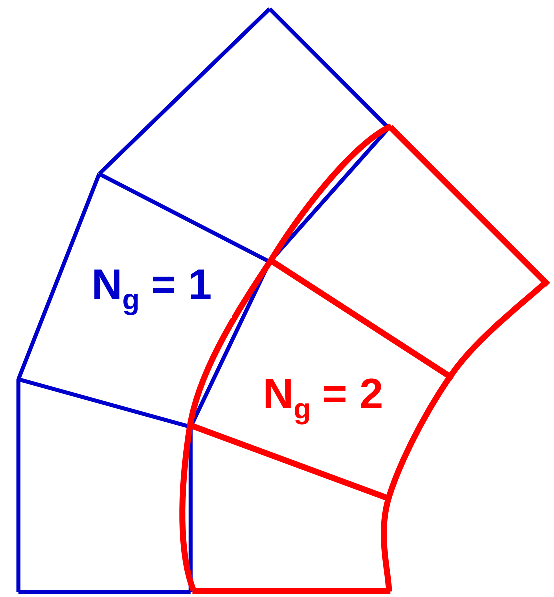


Identical to Interior Scheme

  - Linear Boundary
  - Linear Mappings

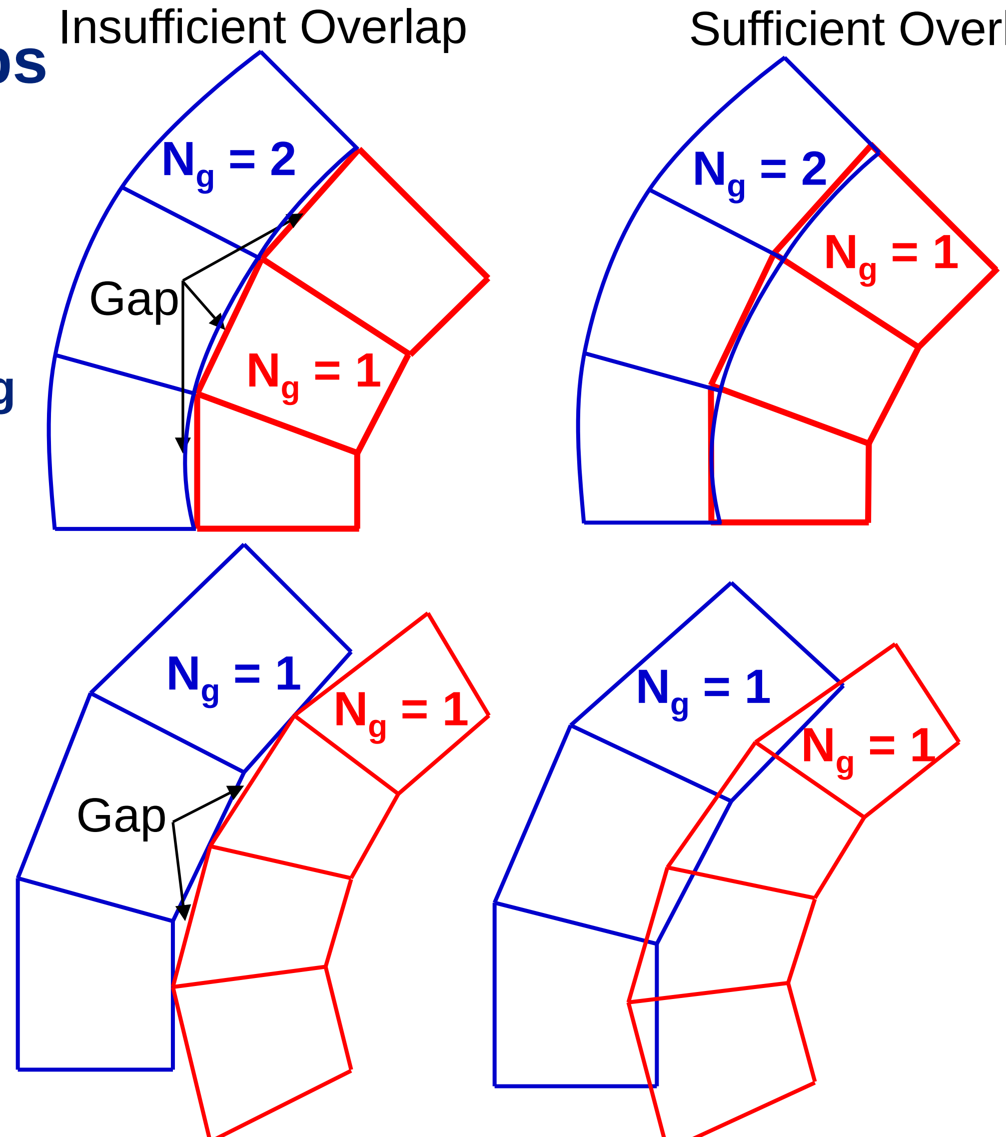


  - Curved Boundary
  - Concave Linear Mapping
  - Convex Quadratic Mapping



DG-Chimera
Orphan Faces

- **Orphans Arise with Gaps Between Mesh Boundaries**
- **Examples:**
  - **Curved Boundary**
  - **Concave Quadratic Mapping**
  - **Convex Linear Mapping**

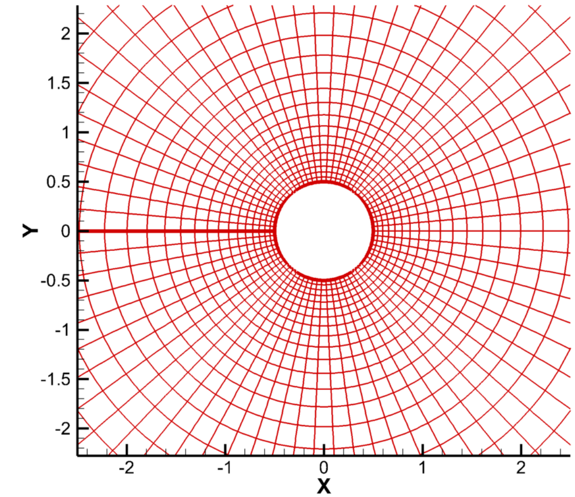


  - **Curved Boundary**
  - **Non-coincident Nodes**
  - **Linear Mappings**

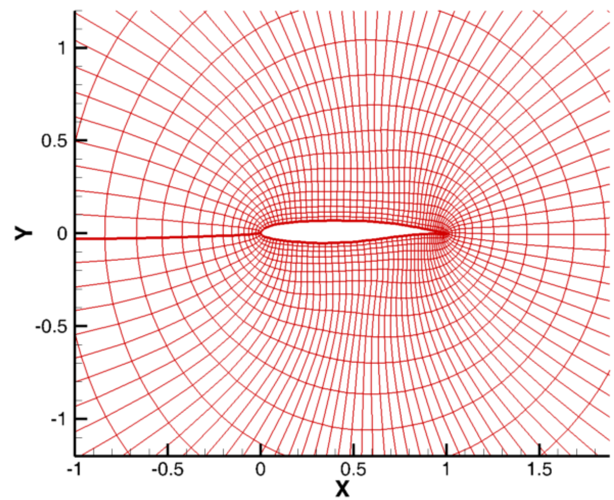
Inviscid Flow Examples

- **Honor of Dr. Benek, Dr. Steger, and Dr. Dougherty**

- **Subsonic Circular Cylinder ($M_\infty = 0.25$)**



- **SKF 1.1 Airfoil ($M_\infty = 0.4$)**



Subsonic Circular Cylinder ($M_\infty = 0.25$) Meshes

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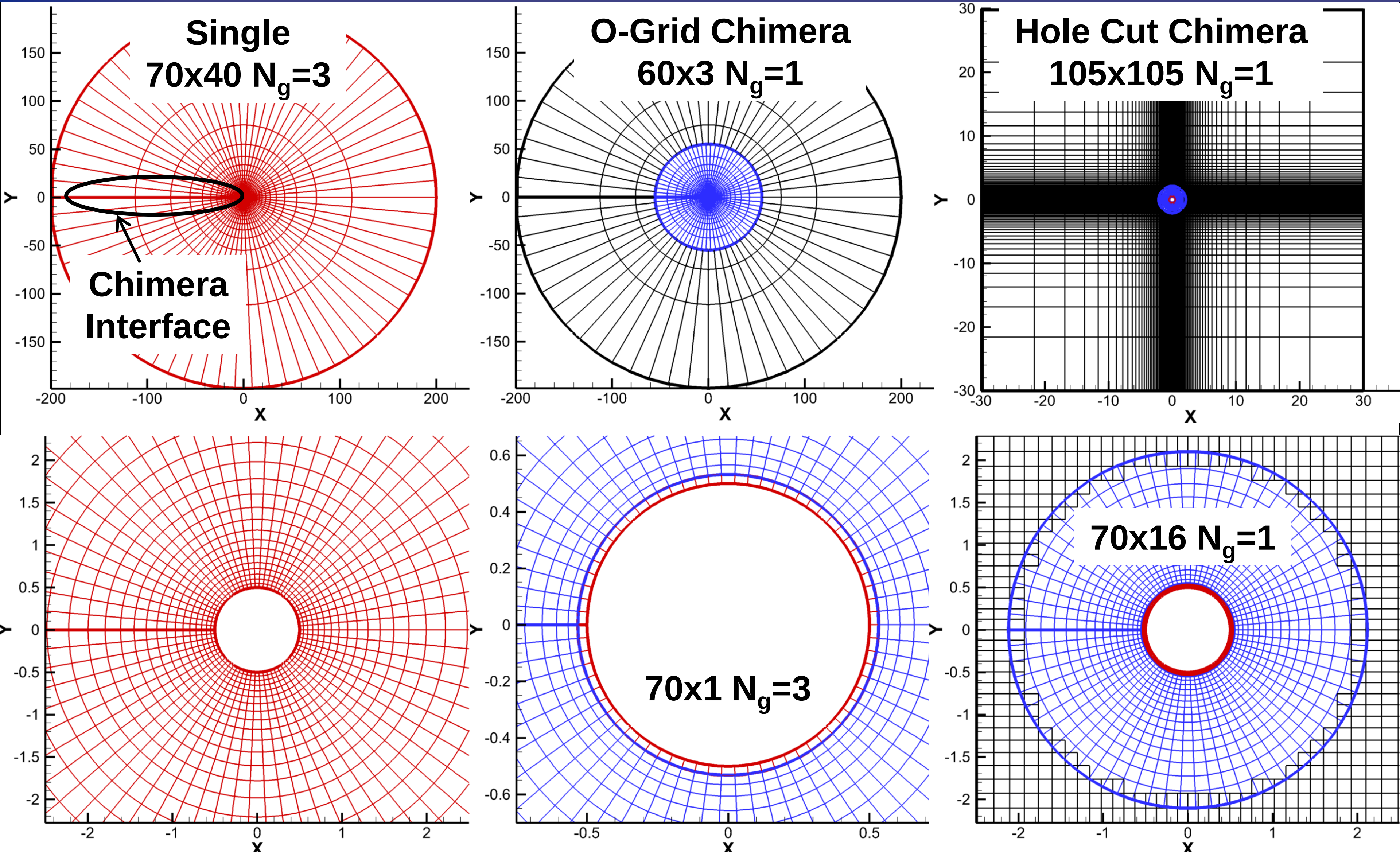

Subsonic Circular Cylinder ($M_\infty = 0.25$) Cp Contour Lines

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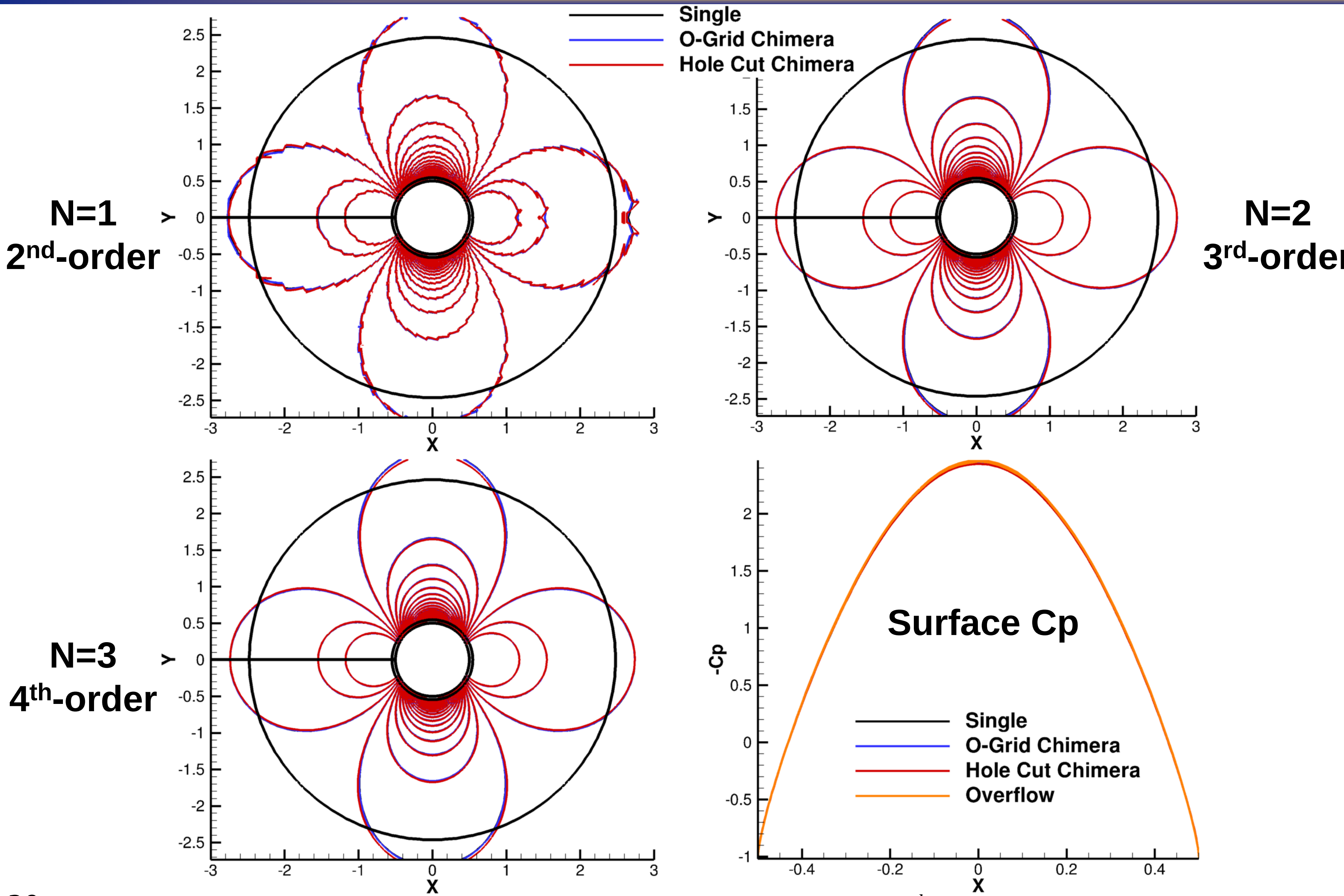

SKF 1.1 Airfoil ($M_\infty = 0.4$) Meshes

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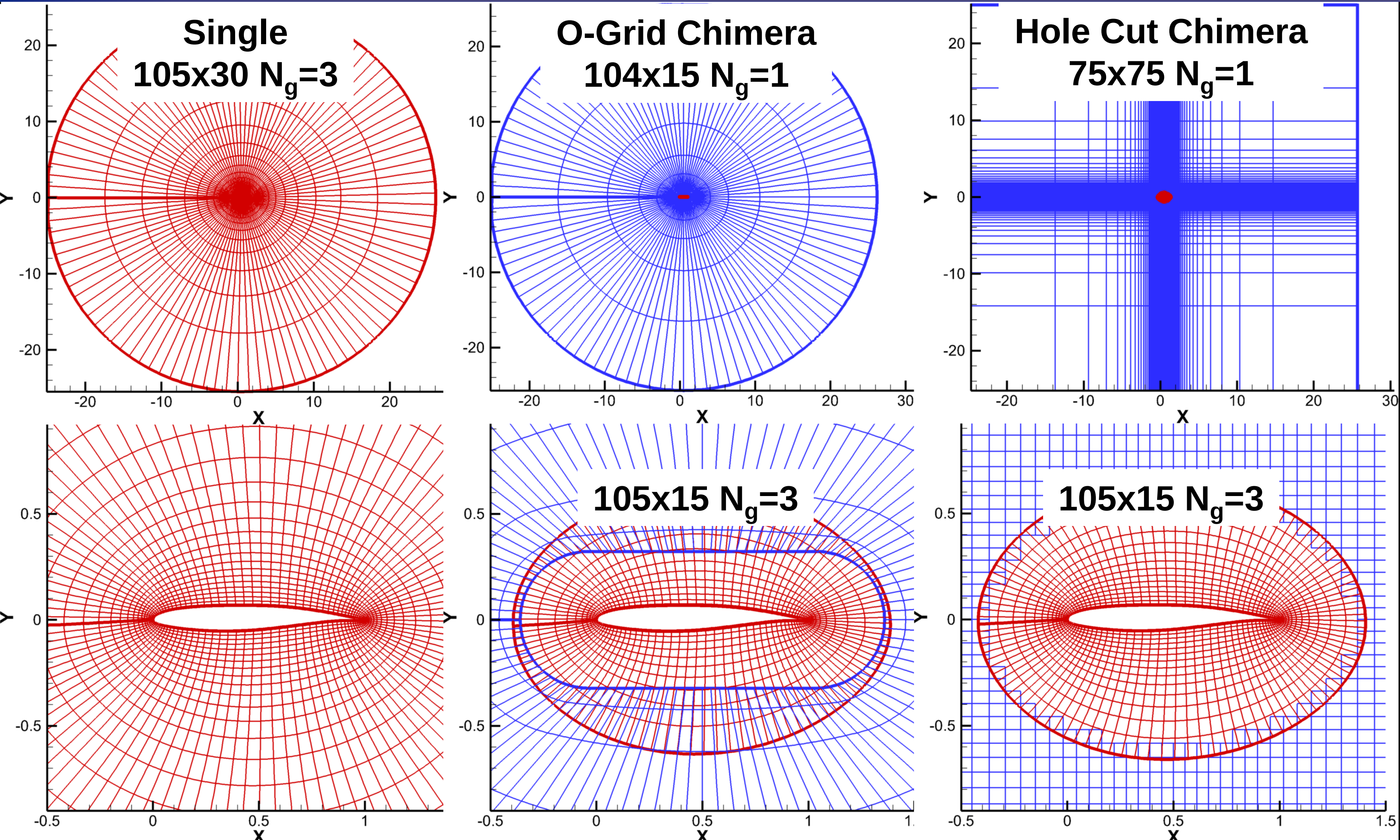
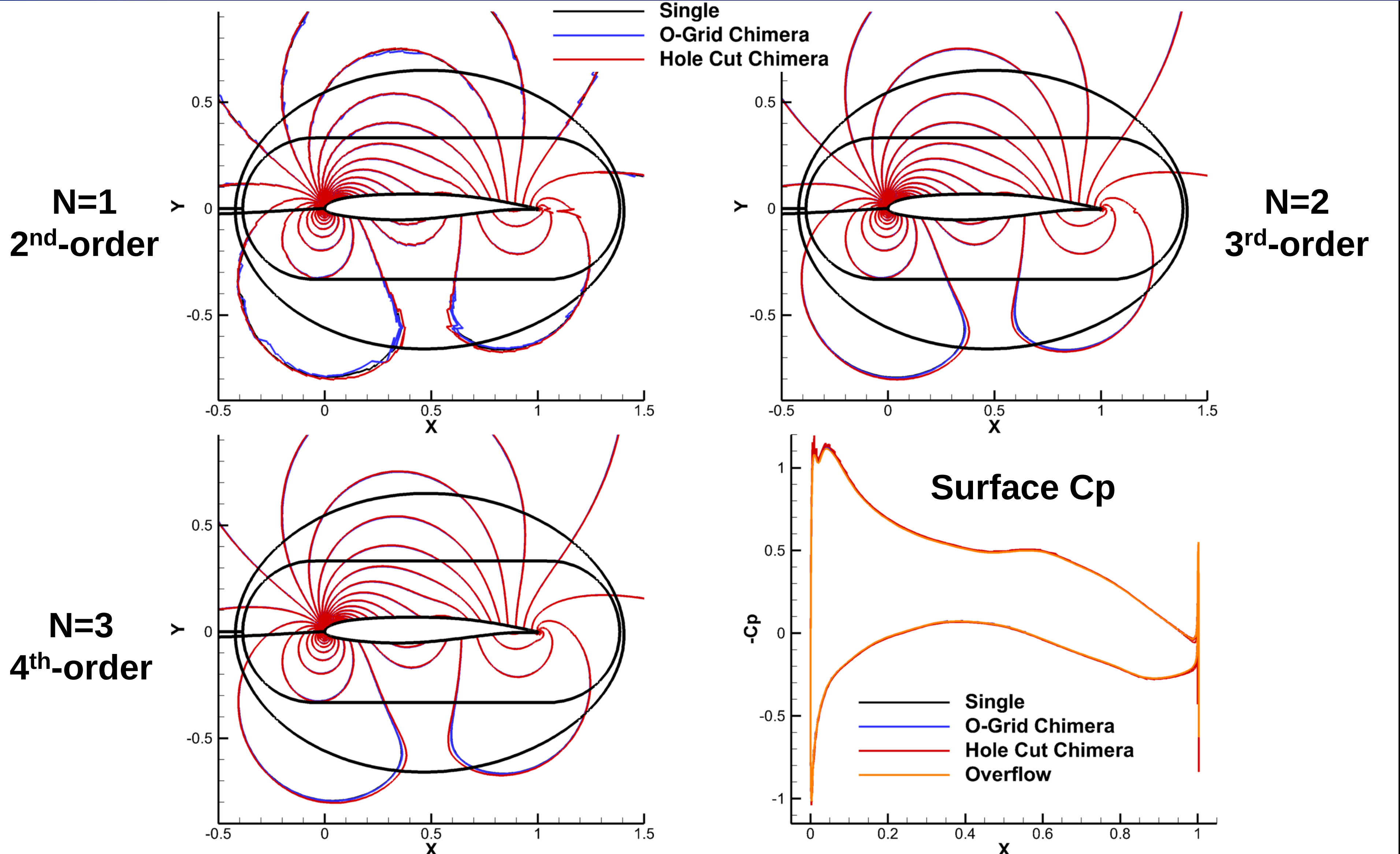


SKF 1.1 Airfoil ($M_\infty = 0.4$)
Cp Contour Lines



SKF 1.1 Airfoil ($M_\infty = 0.4$) Explicit Chimera Convergence

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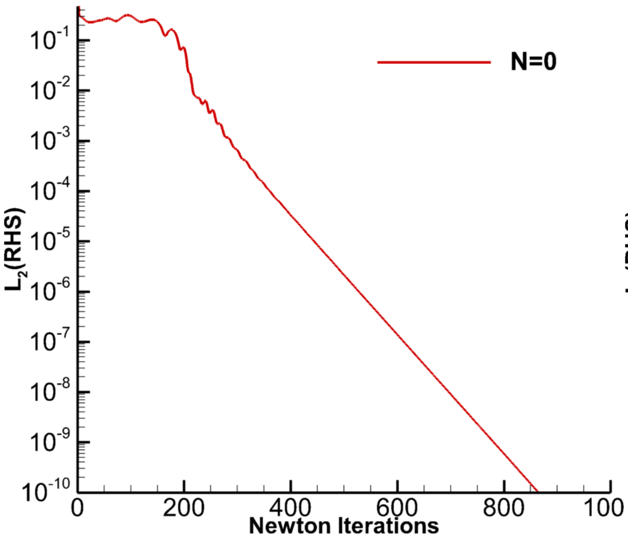
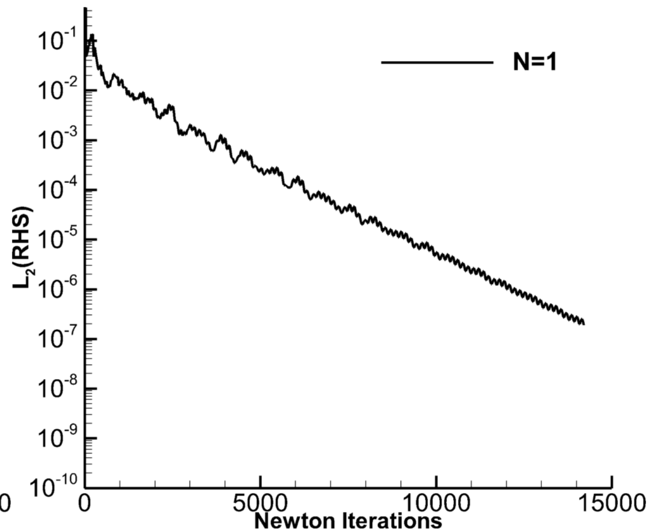
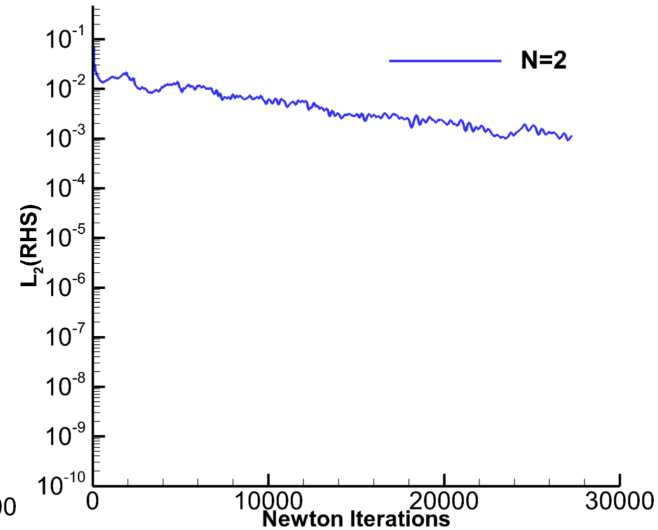
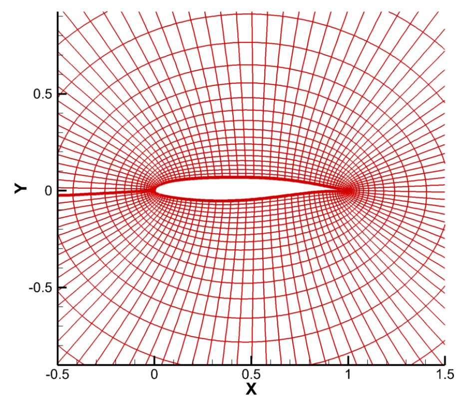
- **Explicit Chimera Boundaries** $A(Q_{Local})\Delta Q = R(Q_{Local}, Q_{Chimera})$

Single Mesh





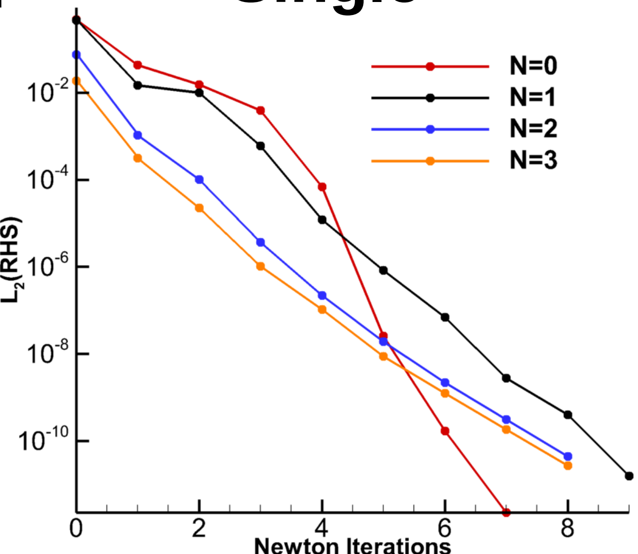


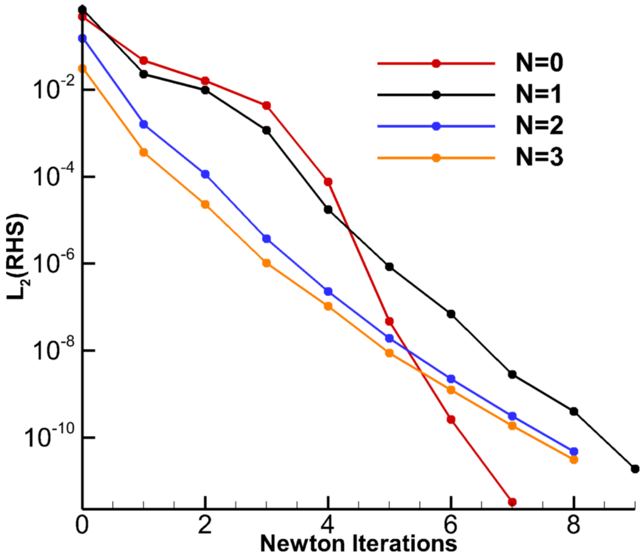


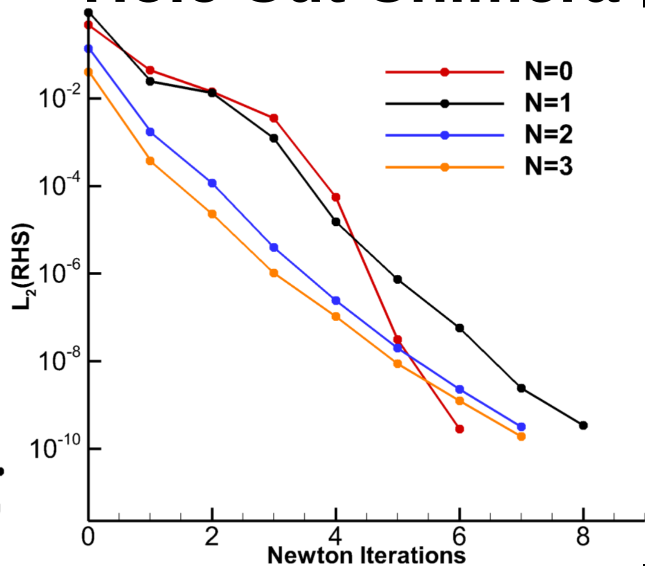
SKF 1.1 Airfoil ($M_\infty = 0.4$)
Implicit Chimera Convergence

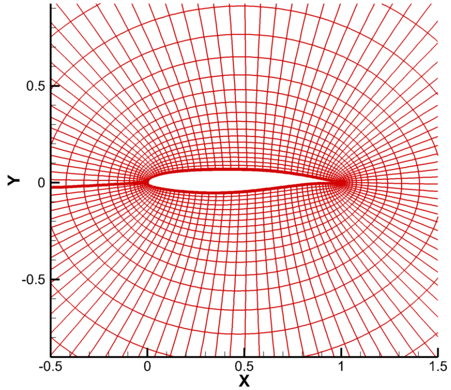
- **Implicit Chimera Boundaries** $A(Q_{Local}, Q_{Chimera})\Delta Q = R(Q_{Local}, Q_{Chimera})$

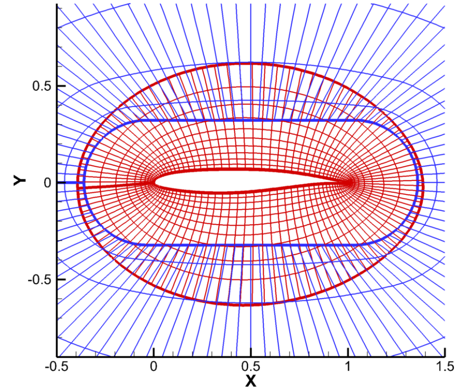
Single | **O-Grid Chimera** | **Hole Cut Chimera**

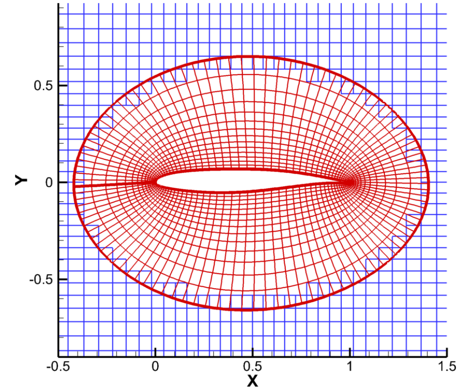












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  - Finite Volume and Fringe Points
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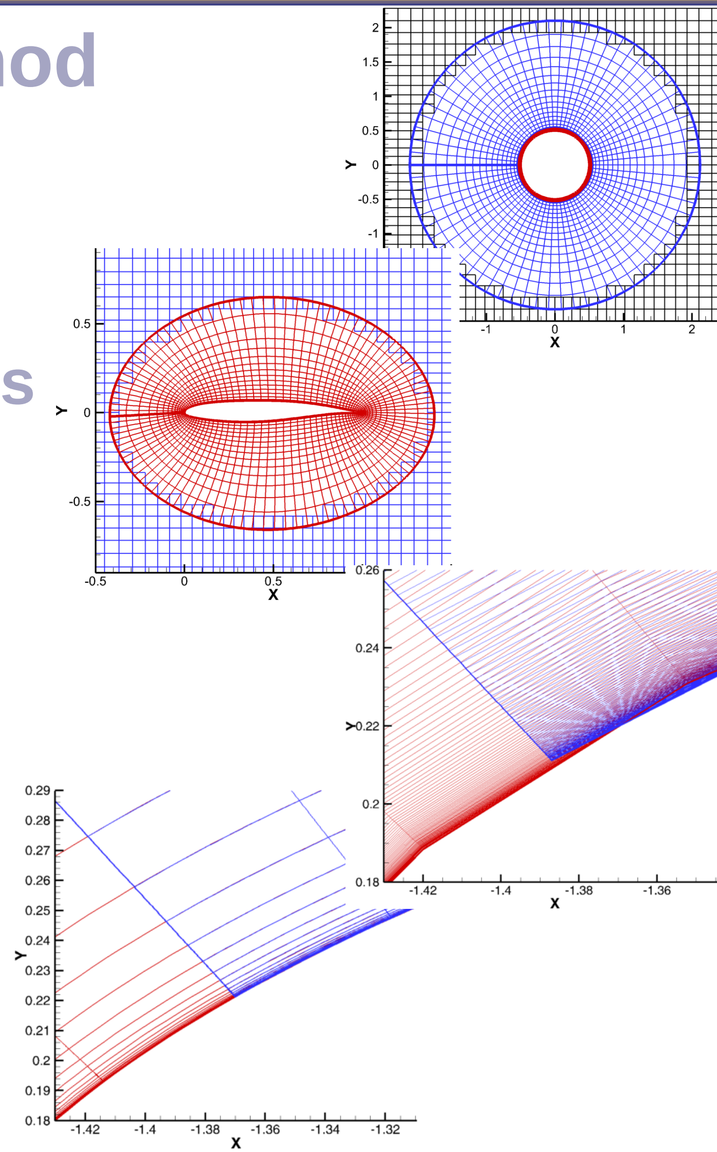
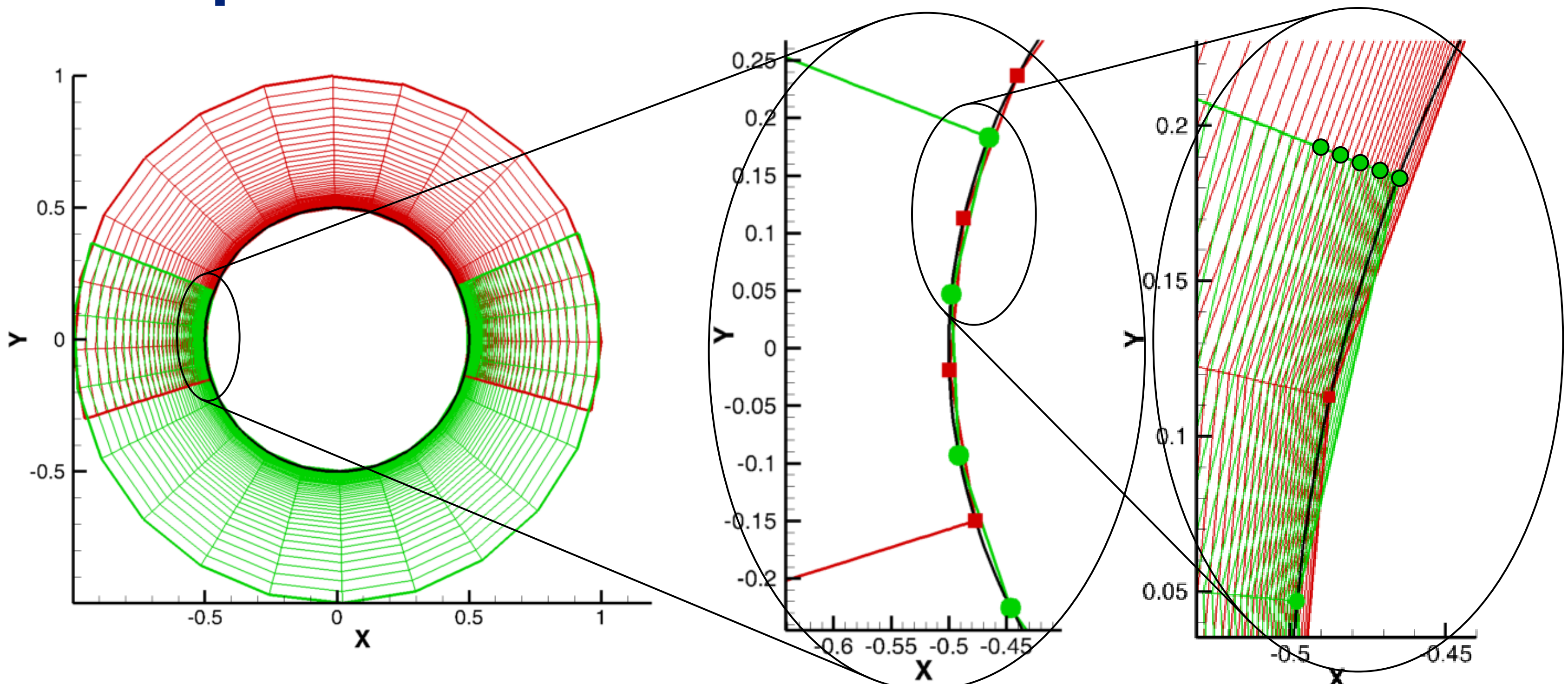
  - **Viscous Flow Examples**
- Conclusion and Future Work



High-Order FD/FV Chimera Overset Regions on Curved Geometry

- **Secant Line Geometric Representation**
- **Interpolation from Incorrect Interior Cells**



- **PEGASUS5 & SUGGAR++: Projection**
- **Discontinuous Galerkin Curved Elements**
  - Elements Follow Surface of Geometry
  - (Dr. Noack Finite Volume)

Viscous Flow Examples

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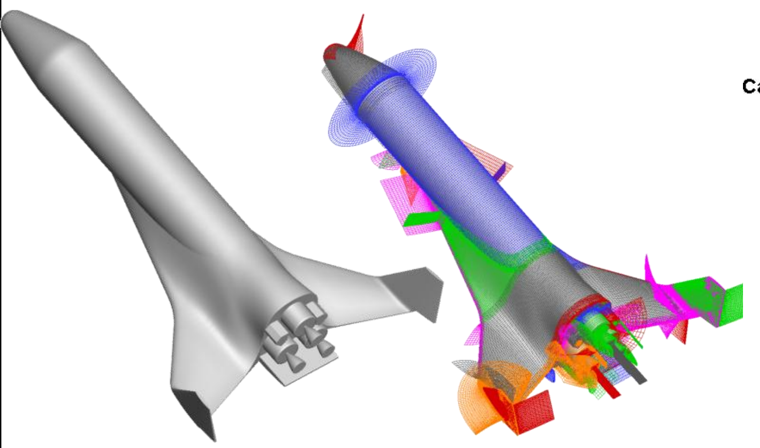
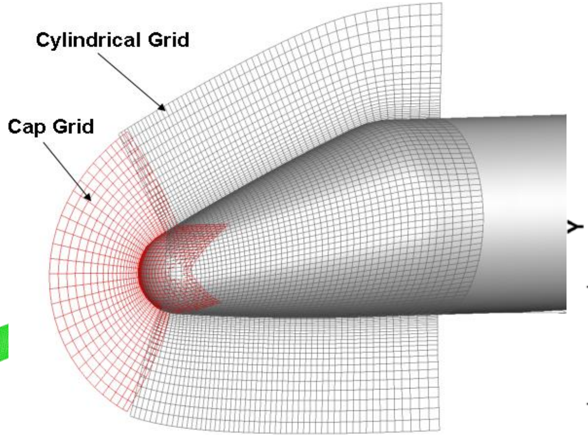
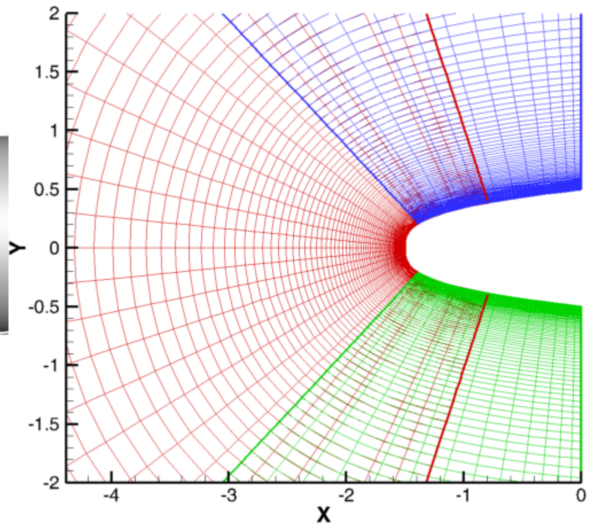
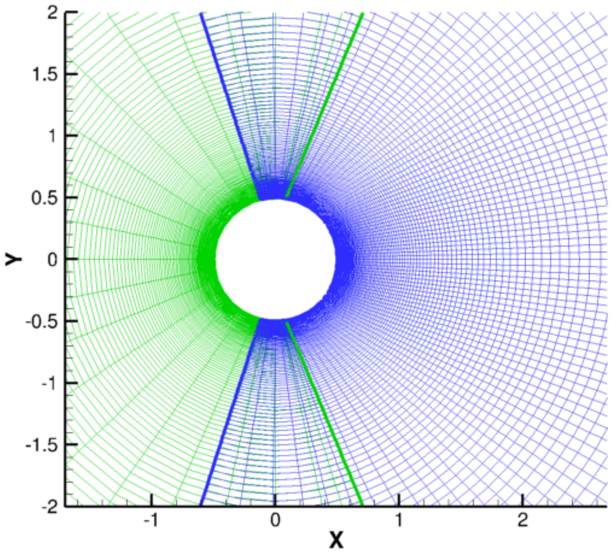
Generic Launch Vehicle







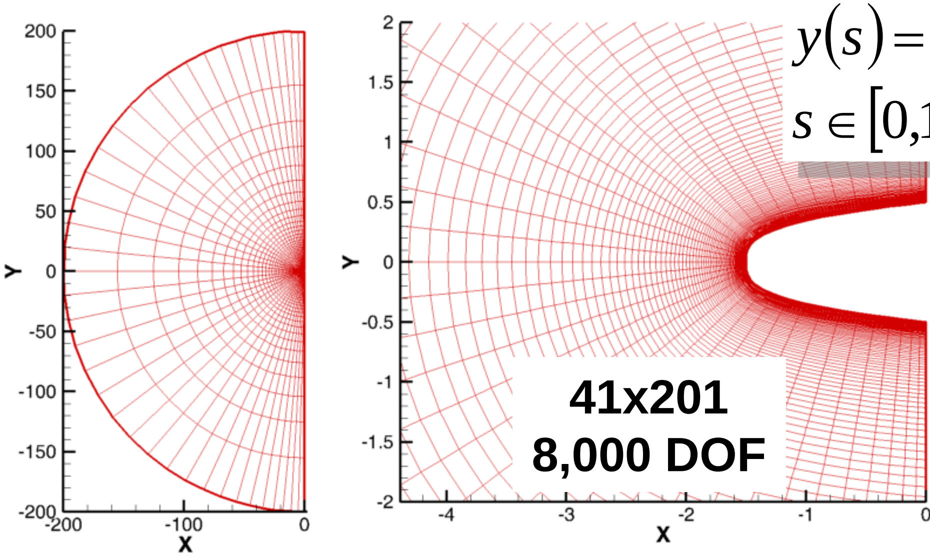
- **Inspired from 3D Overset Mesh**
- **Nose Section**
  - M_∞=0.5, Re=1,000,000
- **Circular Cylinder**
  - M_∞=0.1, Re=40

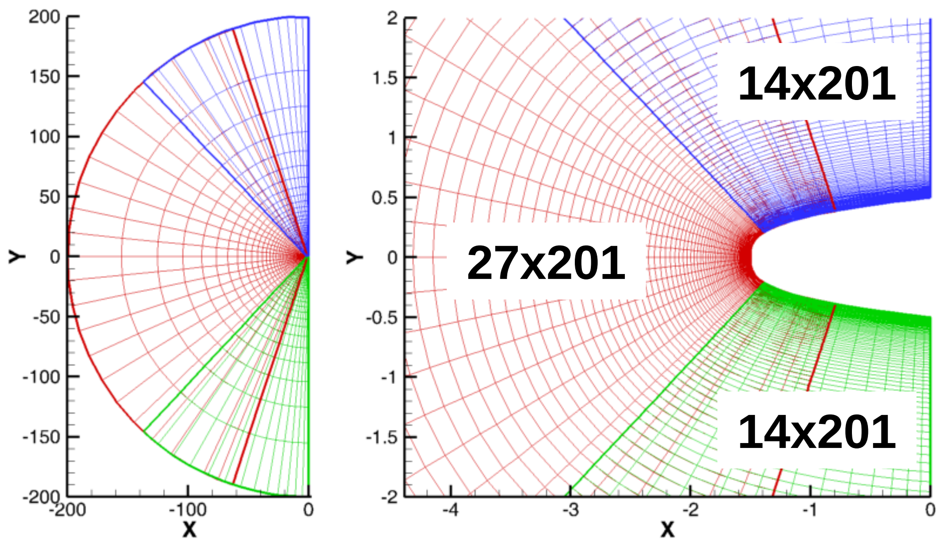


Nose Section Meshes

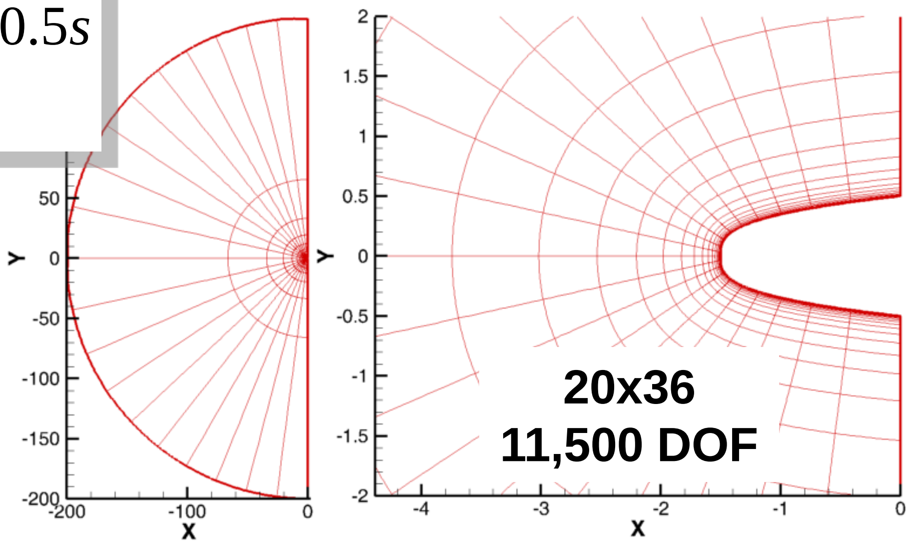
$$x(s) = 1.5s^3$$
$$y(s) = \pm 0.5s$$
$$s \in [0,1]$$

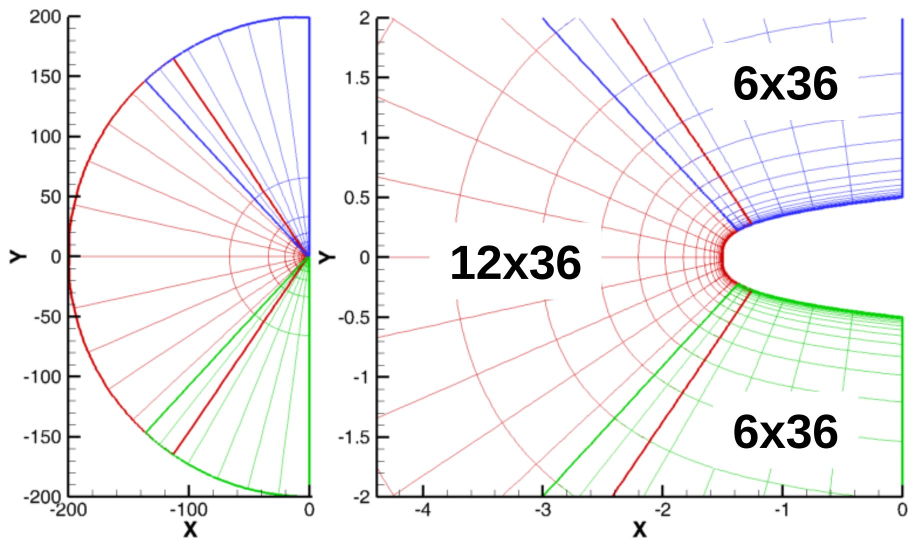
Finite Volume





Discontinuous Galerkin N_g = 3



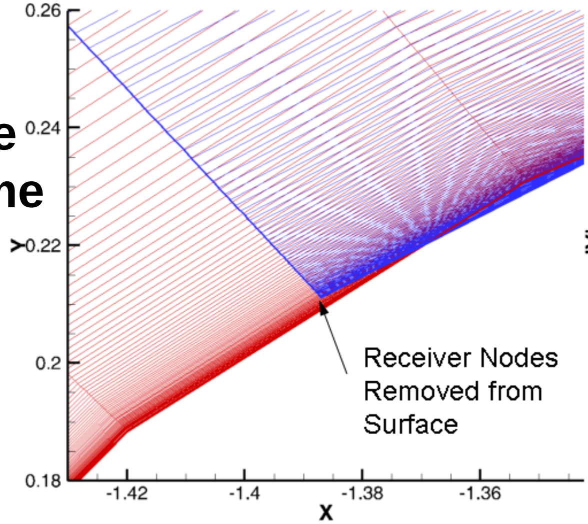


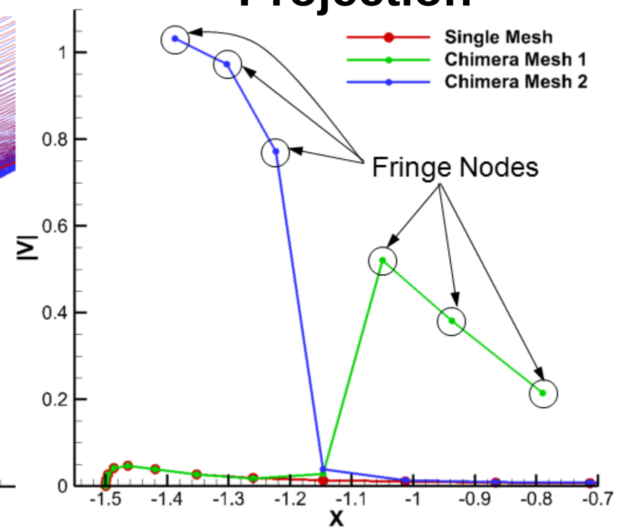
Nose Section Surface Velocity

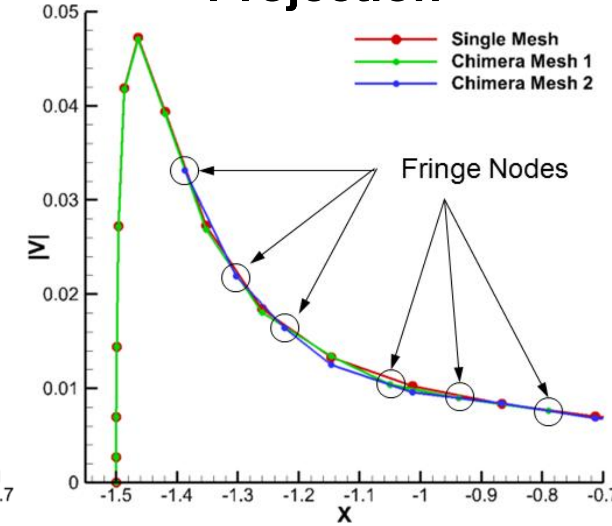
No Projection

With Projection

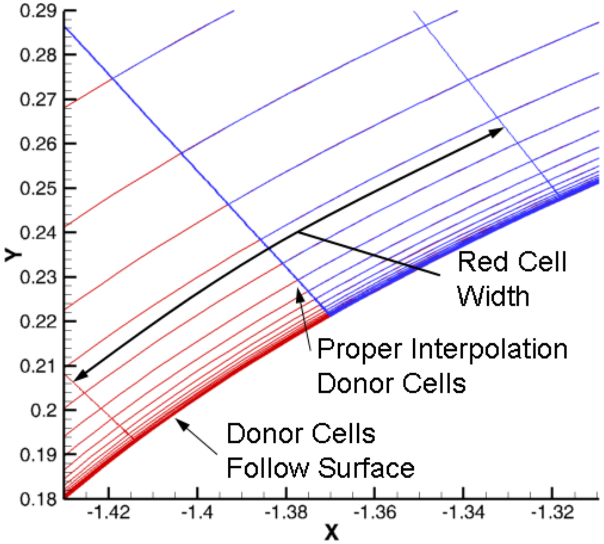
Finite Volume

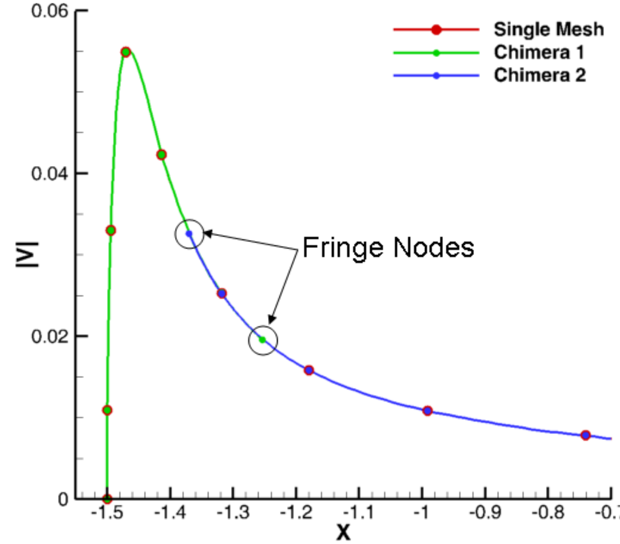






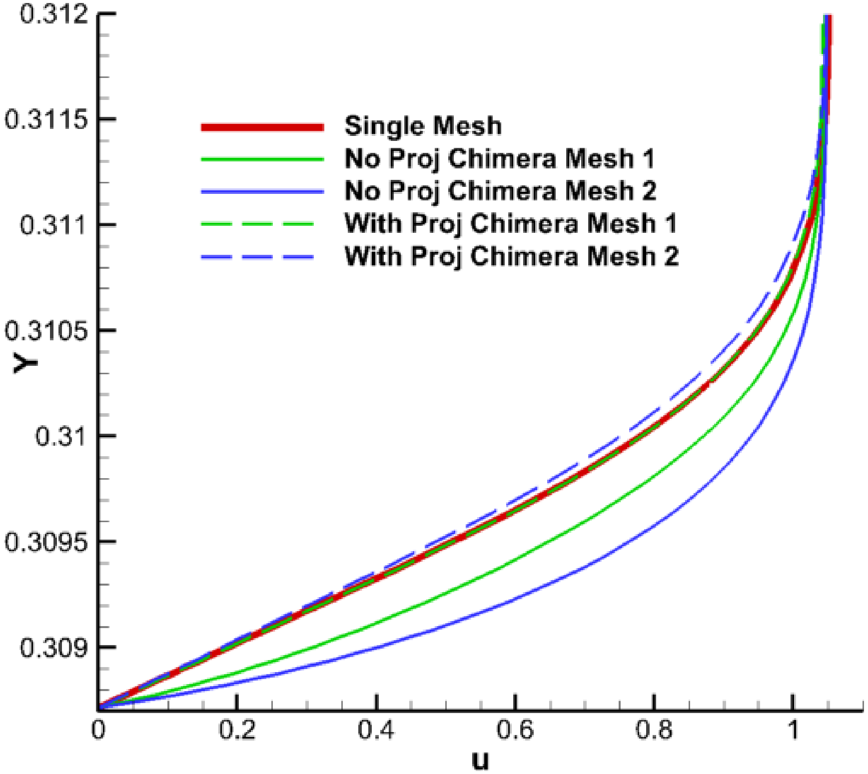
DG

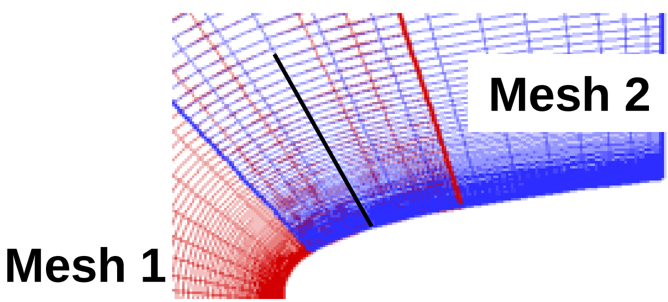




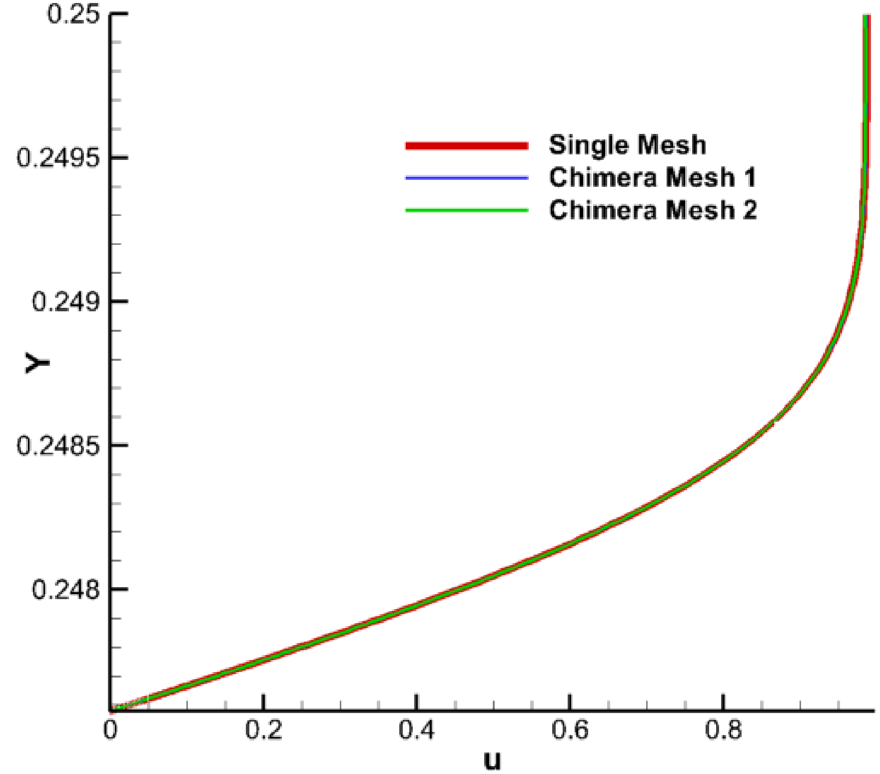
Nose Section
Common Boundary Layer

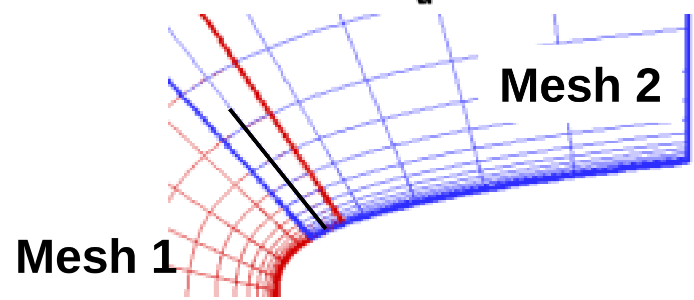
Finite Volume





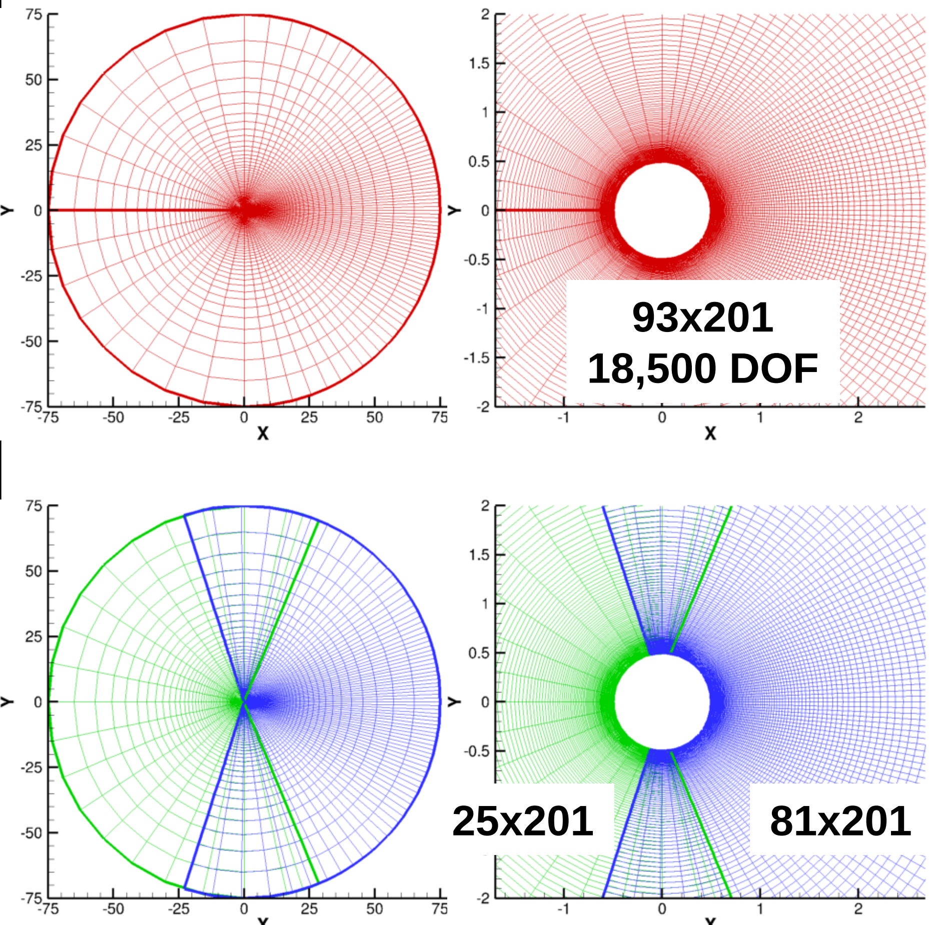
DG



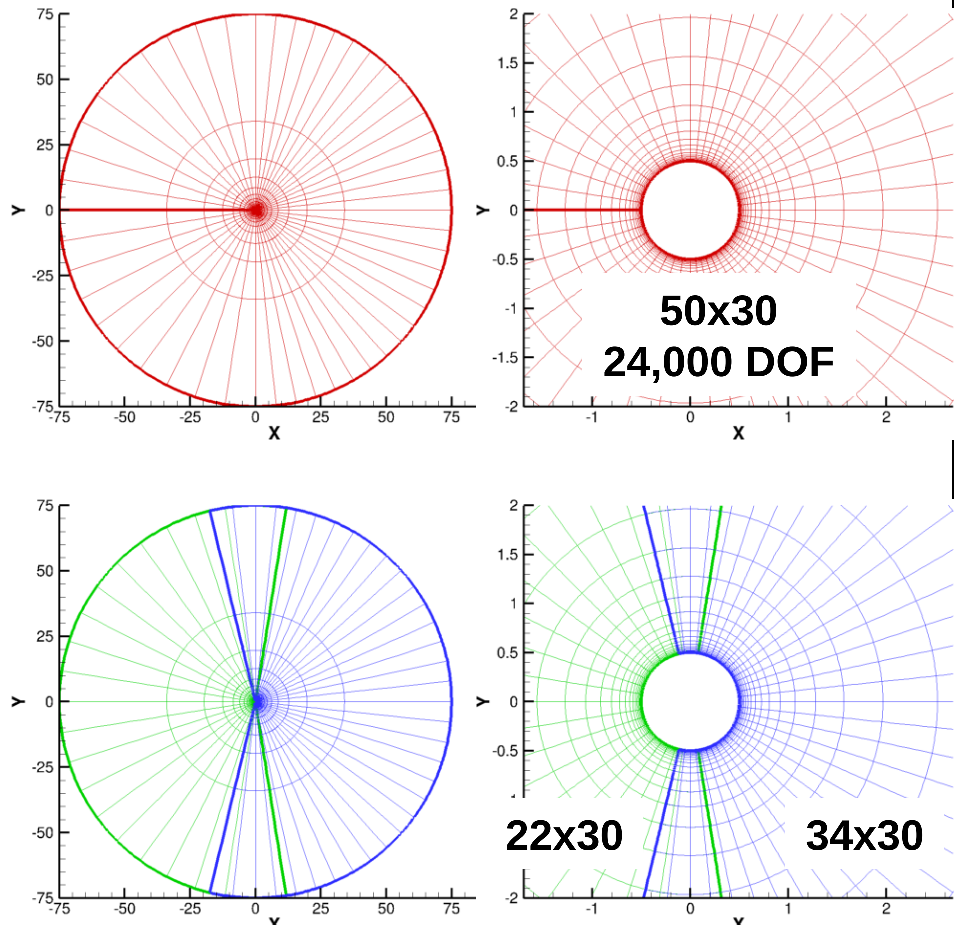


Circular Cylinder Re 40 Meshes

Finite Volume



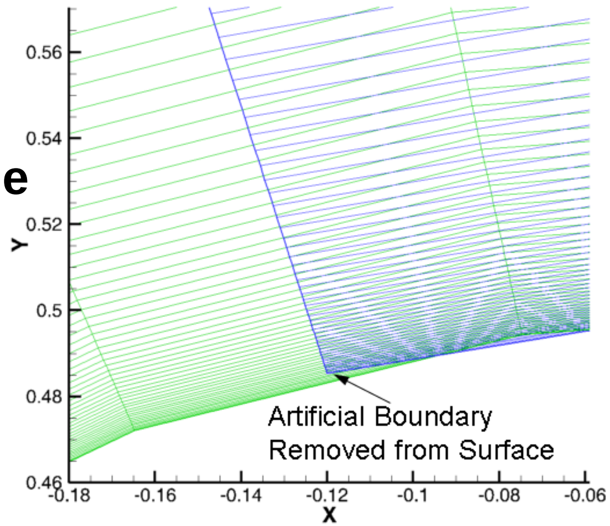
Discontinuous Galerkin $N_g = 3$

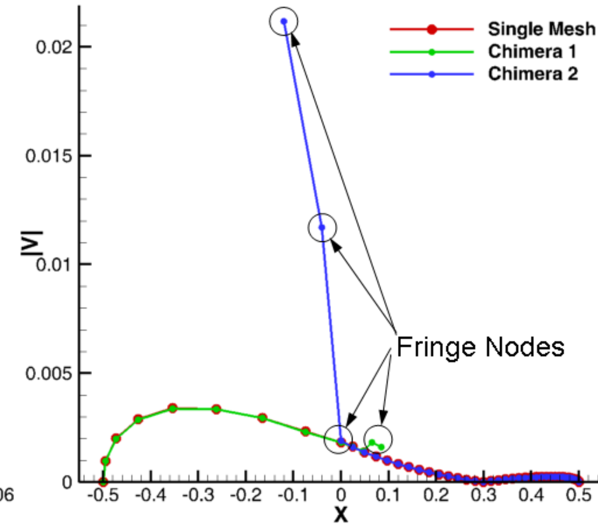


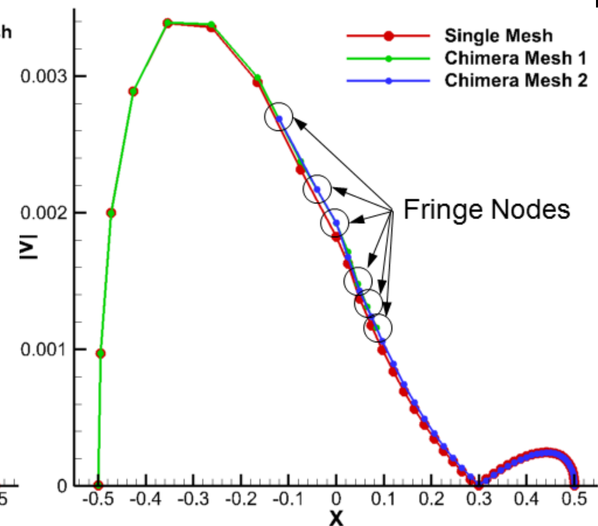
Circular Cylinder Re 40
Surface Velocity

No Projection | **With Projection**

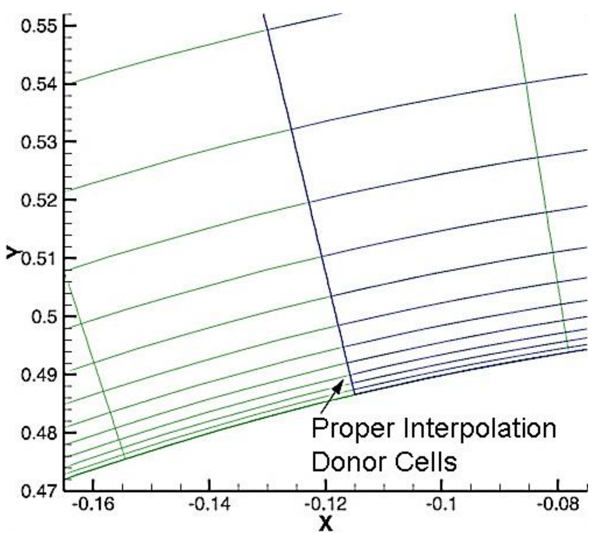
Finite Volume

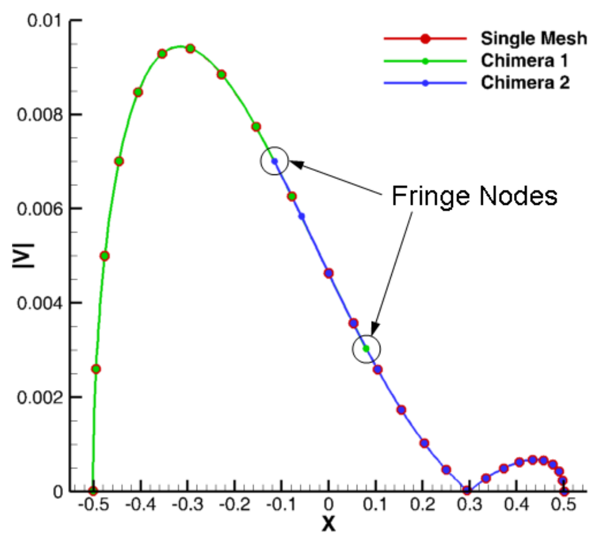






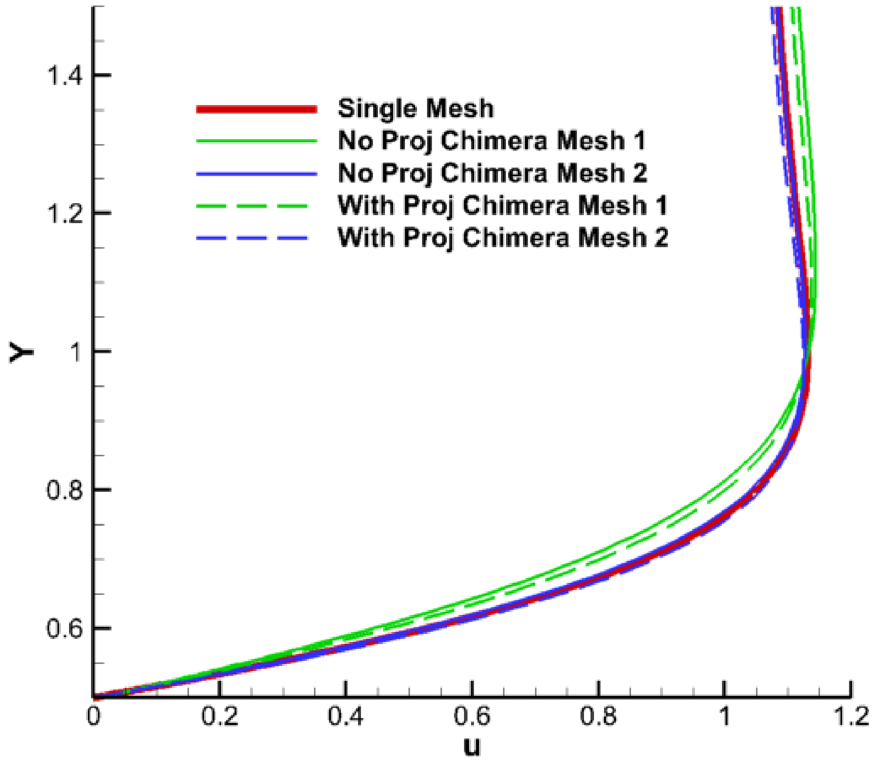
DG



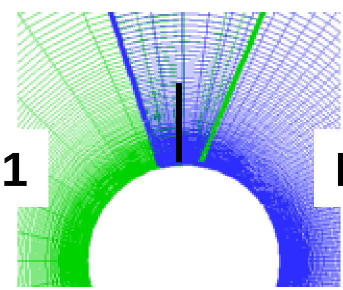


Circular Cylinder Re 40
Common Boundary Layer

Finite Volume

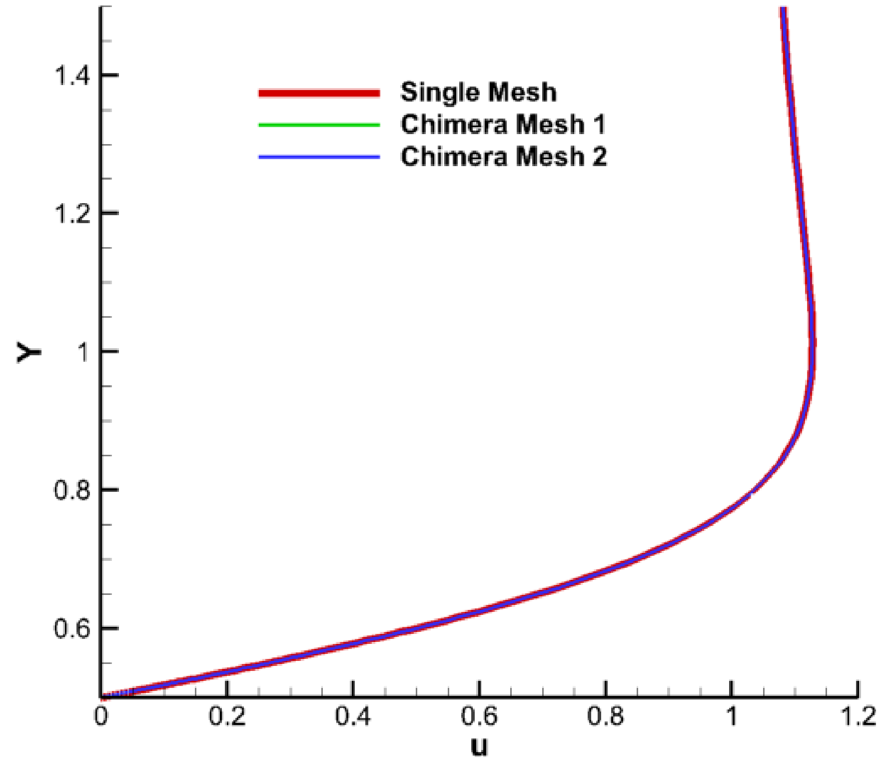


Mesh 1

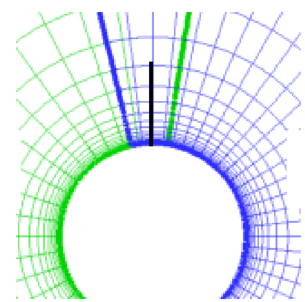


Mesh 2

DG



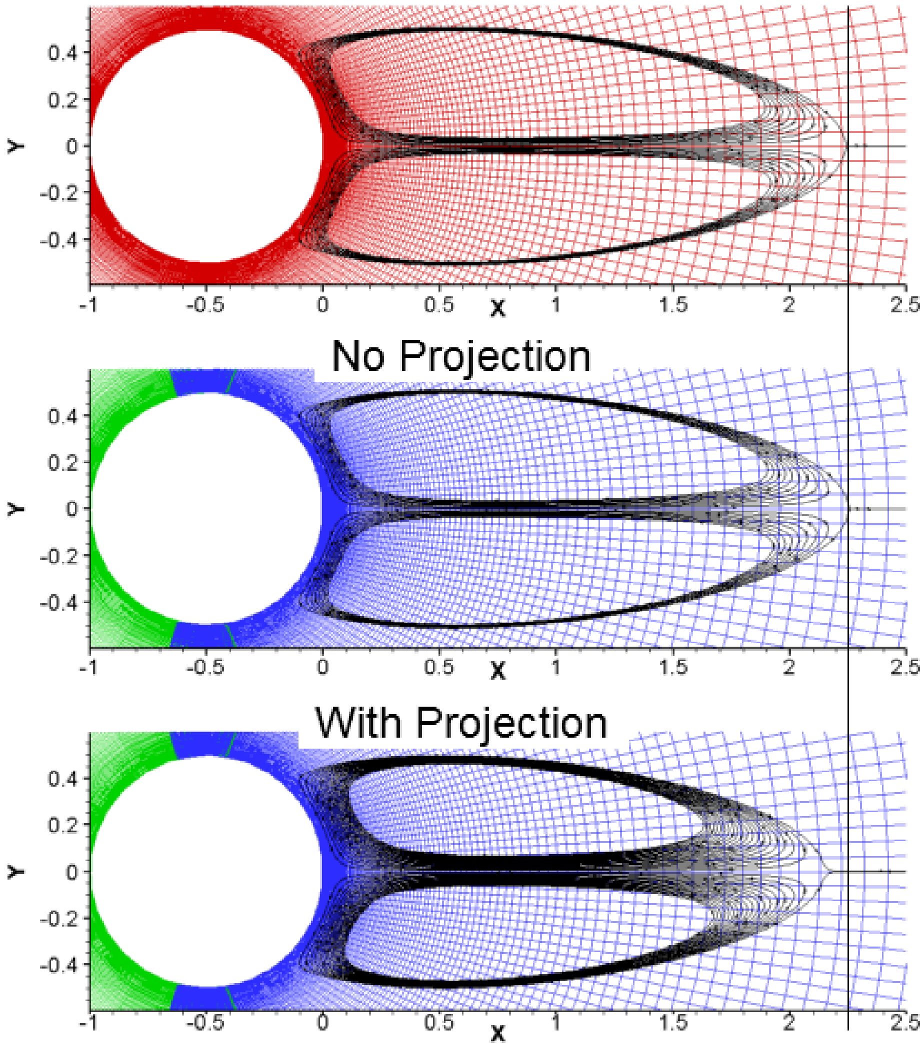
Mesh 1



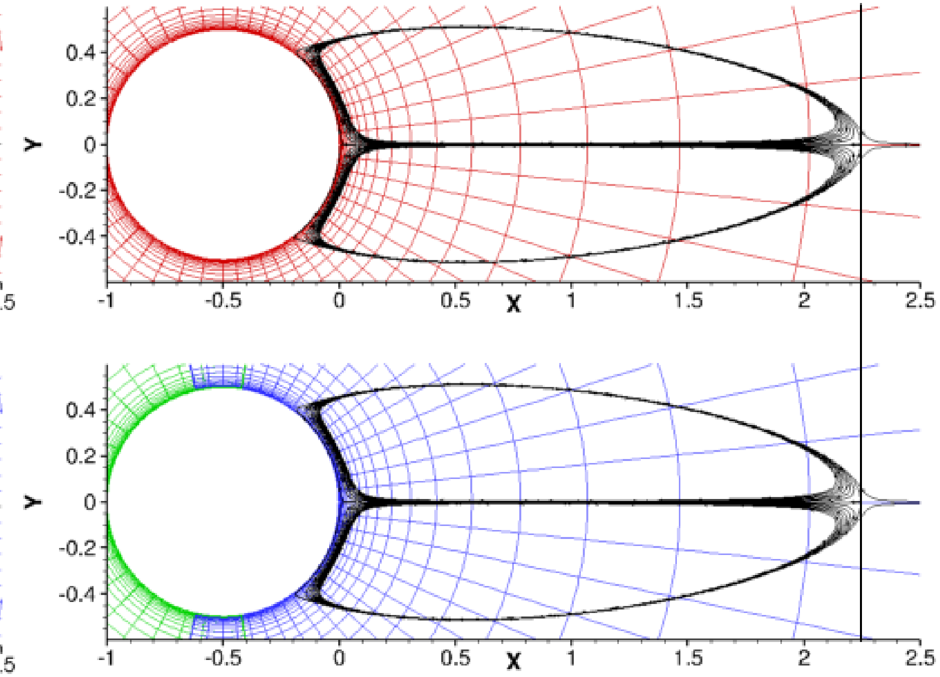
Mesh 2

Circular Cylinder Re 40 Separation Length

Finite Volume



Discontinuous Galerkin



Literature Separation Length: 1.9 < L_s < 2.3

	Single Grid	Chimera Mesh
Finite Volume	2.246	2.260 Without 2.152 With
DG	2.245	2.246

Conclusion and Future Work

- DG-Chimera
  - No Orphan Points Due to Fringe Points
  - Naturally Reduces to Zonal Interface
  - Inherent Proper Interpolation on Curved Geometry
    - Curved Elements
  - Demonstrated on Inviscid/Viscous Flows
- Future Work
  - Extend to 3-D in Space (Mostly complete)
  - Shock Capturing (Mostly Complete)
  - Parallel Execution with MPI

Thank you!
Questions?

This work was supported by the Department of Defense (DoD) through the National Defense Science & Engineering Graduate (NDSEG) Fellowship Program